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Nucleosynthesis and the Early Universe

The cosmic microwave background tells us a great deal about the state of the universe at the time of last scattering ($t_{\rm ls} \approx 0.37\,{\rm Myr}$). However, the opacity of the early universe prevents us from directly seeing what the universe was like at $t < t_{\rm ls}$. Looking at the last scattering surface is like looking at the surface of a cloud, or the surface of the Sun; our curiosity is piqued, and we wish to find out what conditions are like in the opaque regions so tantalizingly hidden from our direct view.

Planck 2016
$z_{rm} \simeq 3371 \pm 23$
$T_0 = 2.7255\,{\rm K}$

Theoretically, many properties of the early universe should be quite simple. For instance, when radiation is strongly dominant over matter, at scale factors $a \ll a_{rm} \approx 2.9 \times 10^{-4}$, or times $t \ll t_{rm} \approx$ 51,880 50 000 yr, the expansion of the universe has the simple power-law form $a(t) \propto t^{1/2}$. The temperature of the blackbody photons in the early universe, which decreases as $T \propto a^{-1}$ as the universe expands, is given by the convenient relation

(2.41) $T(z) = T_0(1+z) = \frac{T_0}{a(t)}$
(5.60) $a(t) \propto (t/t_0)^{1/2}$
RADIATION-DOM

$$T(t) \approx 10^{10}\,{\rm K}\left(\frac{t}{1\,{\rm s}}\right)^{-1/2}, \qquad (9.1)$$

or equivalently

$$kT(t) \approx 1\,{\rm MeV}\left(\frac{t}{1\,{\rm s}}\right)^{-1/2}. \qquad (9.2)$$

Thus the mean energy per photon was

(8.19) $T_{rec} = \frac{Q}{2.7k} = \frac{13.6\,{\rm eV}}{2.7k}$

$$E_{\rm mean}(t) \approx 2.7kT(t) \approx 3\,{\rm MeV}\left(\frac{t}{1\,{\rm s}}\right)^{-1/2}. \qquad (9.3)$$

The Large Hadron Collider (LHC), on the border between France and Switzerland, accelerates protons to an energy $E = 7 \times 10^6\,{\rm MeV} = 7\,{\rm TeV}$, about 7500 times the rest energy of a proton. The LHC is a remarkable piece of engineering; with a main ring 27 kilometers in diameter, it is sometimes called the largest single machine in the world. However, when the universe had an

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age of $t \approx 2 \times 10^{-13}$ s, the average, run-of-the-mill particle energy was equal to that attained by the LHC. Thus, the early universe is referred to as "the poor man's particle accelerator," since it provided particles of very high energy without running up an enormous electricity bill or requiring billions of euros in funding.

9.1 Nuclear Physics and Cosmology

As the universe has expanded and cooled, the mean energy per photon has dropped from $E_{\text{mean}}(t_P) \sim E_P \sim 10^{28}$ eV at the Planck time to $E_{\text{mean}}(t_0) \sim 10^{-3}$ eV at the present day. Thus, by studying the universe as it expands, we sample 31 orders of magnitude in particle energy. Within this wide energy range, some energies are of more interest than others to physicists. For instance, to physicists studying recombination and photoionization, the most interesting energy scale is the ionization energy of an atom. The ionization energy of hydrogen is $Q = 13.6$ eV, as we have already noted. The ionization energies of other elements (that is, the energy required to remove the most loosely bound electron in the neutral atom) are roughly comparable; they range from 24.6 eV for helium to 4 eV for heavy alkali metals like cesium. Thus, atomic physicists, when considering the ionization of atoms, typically deal with energies of ~ 10 eV, in round numbers.

Nuclear physicists are concerned not with ionization and recombination (removing or adding electrons to an atom), but with the much higher energy processes of fission and fusion (splitting or merging atomic nuclei). An atomic nucleus contains Z protons and N neutrons, where $Z \geq 1$ and $N \geq 0$. Protons and neutrons are collectively called *nucleons*. The total number of nucleons within an atomic nucleus is called the *mass number*, and is given by the formula $A = Z+N$. The proton number Z of a nucleus determines the atomic element to which that nucleus belongs. For instance, hydrogen (H) nuclei all have $Z = 1$, helium (He) nuclei have $Z = 2$, lithium (Li) nuclei have $Z = 3$, beryllium (Be) nuclei have $Z = 4$, and so on, through the complete periodic table. Although all atoms of a given element have the same number of protons in their nuclei, different isotopes of an element can have different numbers of neutrons. A particular isotope of an element is designated by prefixing the mass number A to the symbol for that element. For instance, a standard hydrogen nucleus, with one proton and no neutrons, is symbolized as ^{1}H. (Since an ordinary hydrogen nucleus is nothing more than a proton, we may also write p in place of ^{1}H when considering nuclear reactions.) Heavy hydrogen, or *deuterium*, contains one proton and one neutron, and is symbolized as ^{2}H. (Since the deuterium nucleus is mentioned frequently in the context of nuclear fusion, it has its own name, the "deuteron," and its own special symbol, D.) Ordinary helium contains two protons and two neutrons, and is symbolized as ^{4}He.

$N_p + N_n$
$\downarrow \quad \downarrow$
$A = Z + N$

$Z + N = A$
H: $1 + 0 = 1$
D: $1 + 1 = 2$
^{3}He: $2 + 1 = 3$
^{4}He. $2 + 2 = 4$
etc

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$E_S = \frac{E_B}{A} = \frac{E_B}{Z+N}$

The binding energy B of a nucleus is the energy required to pull it apart into its component protons and neutrons. Equivalently, it is the energy released when a nucleus is fused together from individual protons and neutrons. For instance, when a neutron and a proton are bound together to form a deuteron, an energy of $B_D = 2.22\,\text{MeV}$ is released:

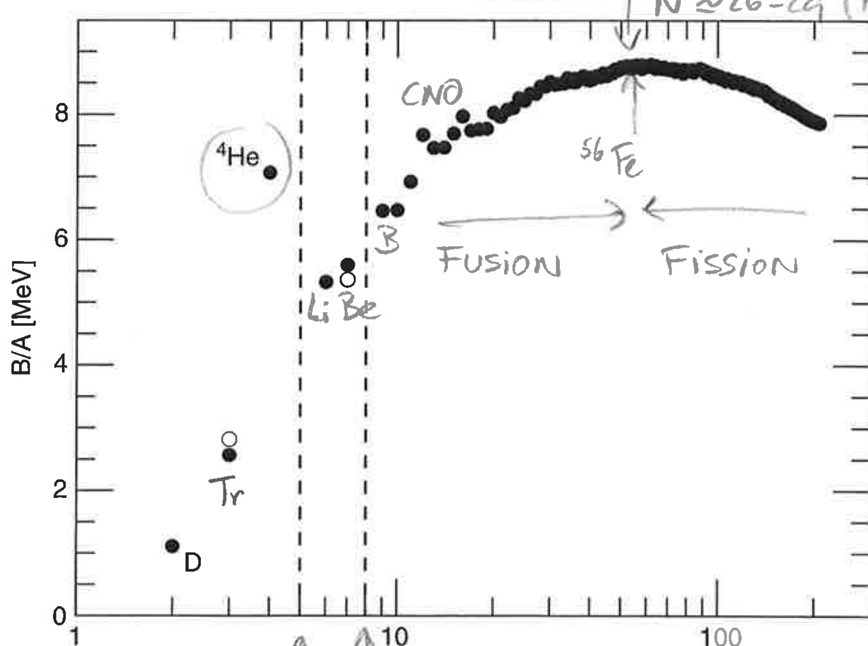
$$p + n \rightleftharpoons \text{D} + 2.22\,\text{MeV}. \tag{9.4}$$

The deuteron is not very tightly bound, compared to other atomic nuclei. Figure 9.1 plots the binding energy per nucleon (B/A) for atomic nuclei with different mass numbers. Note that ^{4}He, with a total binding energy of $B = 28.30\,\text{MeV}$, and a binding energy per nucleon of $B/A = 7.07\,\text{MeV}$, is relatively tightly bound, compared to other light nuclei (that is, nuclei with $A \leq 10$). The most tightly bound nuclei are those of ^{56}Fe, ^{58}Fe, and ^{62}Ni, which all have $B/A \approx 8.79\,\text{MeV}$. Thus, nuclei more massive than iron or nickel can release energy by fission – splitting into lighter nuclei. Nuclei less massive than iron or nickel can release energy by fusion – merging into heavier nuclei.

Just as studies of ionization and recombination deal with an energy scale of $\sim 10\,\text{eV}$ (a typical ionization energy), so studies of nuclear fusion and fission deal with an energy scale of $\sim 8\,\text{MeV}$ (a typical binding energy per nucleon). Moreover, just as electrons and protons combined to form neutral hydrogen atoms when the temperature dropped sufficiently far below the ionization energy of hydrogen ($Q = 13.6\,\text{eV}$), so protons and neutrons must have fused to form deuterons when the temperature dropped sufficiently far below the binding energy

$T_{H\text{-}ion} \simeq 3\times10^4\,\text{K}$

Peak at $A \simeq 56$–62, $N \simeq 26$–29 (Fe-Ni)



Note absence of $A \simeq 5$ (>He) and $A \simeq 8$ (Be) $\Leftrightarrow$ Don't form ^{5}Li, ^{8}Be, ^{8}B easily

Figure 9.1 Binding energy per nucleon (B/A) as a function of the number of nucleons (A). Stable isotopes are shown as solid dots; the open dots represent the isotopes ^{3}H and of deuterium ($B_D = 2.22$ MeV). The epoch of recombination was thus preceded by an epoch of nuclear fusion, commonly called the epoch of Big Bang nucleosynthesis (BBN). Nucleosynthesis in the early universe starts by the fusion of neutrons and protons to form deuterons, then proceeds to form heavier nuclei by successive acts of fusion. Since the binding energy of deuterium is larger than the ionization energy of hydrogen by a factor $B_D/Q = 1.6 \times 10^5$, we would expect, as a rough estimate, the synthesis of deuterium to occur at a temperature 1.6×10^5 times higher than the recombination temperature $T_{rec} = 3760$ K. That is, deuterium synthesis occurred at a temperature $T_{nuc} \approx 1.6 \times 10^5 (3760\,\mathrm{K}) \approx 6 \times 10^8$ K, corresponding to a time $t_{nuc} \approx 300$ s. This estimate, as we'll see when we do the detailed calculations, gives a temperature slightly too low, but it certainly gives the right order of magnitude. As indicated in the title of Steven Weinberg's classic book, *The First Three Minutes*, the entire saga of Big Bang nucleosynthesis takes place when the universe is only a few minutes old.

$z_{rec} \simeq 1380$

10^9 K

← FIRST 5 min

One thing we can say about Big Bang nucleosynthesis, after taking a look at the present-day universe, is that it was shockingly inefficient. From an energy viewpoint, the preferred universe would be one in which the baryonic matter consisted of an iron-nickel alloy. Obviously, we do not live in such a universe. Currently, three-fourths of the baryonic component (by mass) is still in the form of unbound protons, or ^{1}H. Moreover, when we look for nuclei heavier than ^{1}H, we find that they are primarily ^{4}He, a relatively lightweight nucleus; iron and nickel provide only 0.15% of the baryonic mass of our galaxy. The primordial helium fraction of the universe (that is, the helium fraction before nucleosynthesis begins in stars) is usually expressed as the dimensionless number

$$Y_p \equiv \frac{\rho(^4\mathrm{He})}{\rho_{\mathrm{bary}}} \simeq 0.2467 \pm .0002 \quad \text{Planck (2016)} \tag{9.5}$$

That is, Y_p is the mass density of ^{4}He divided by the mass density of all the baryonic matter. The outer regions of the Sun have a helium mass fraction $Y = 0.27$. However, the Sun is made of recycled interstellar gas, which was contaminated by helium formed in earlier generations of stars. When we look at astronomical objects of different sorts, we find a minimum value of $Y = 0.24$. That is, baryonic objects such as stars and gas clouds are all at least 24 percent helium by mass.[1]

9.2 Neutrons and Protons

The basic building blocks for nucleosynthesis are neutrons and protons. The rest energy of a neutron is greater than that of a proton by an amount

$$Q_n = m_n c^2 - m_p c^2 = 1.29\ \mathrm{MeV} \simeq 2.5\, m_e c^2 \tag{9.6}$$

$939.5 - 938.2$

[1] Condensed objects that have undergone chemical or physical fractionation can be much lower in helium than this value. For instance, your helium fraction is $\ll 24\%$.

A free neutron is unstable. It decays through the emission of an electron and an electron antineutrino,

$$\boxed{n \to p^{+} + e^- + \bar{\nu}_e,} \quad \text{NEUTRON DECAY} \quad \tau_n \simeq 15\ \text{min} \tag{9.7}$$

with a decay time $\tau_n \overset{\simeq 15\ \text{min}}{=} 880\,\text{s}$. That is, if you start with a population of free neutrons, after a time t, a fraction $f = \exp(-t/\tau_n)$ will remain.[2] Since the energy Q_n released by the decay of a neutron into a proton is greater than the rest energy of an electron ($m_e c^2 = 0.51\,\text{MeV}$), the remainder of the energy is carried away by the kinetic energy of the electron and the energy of the electron antineutrino. With a decay time of only fifteen minutes, the existence of a free neutron is as fleeting as fame; once the universe was several hours old, it contained essentially no free neutrons. However, a neutron bound into a stable atomic nucleus is preserved against decay. Neutrons are still around today because they've been tied up in deuterium, helium, and other atoms. $m_e c^2 \simeq 511$ KeV

Let's consider the state of the universe when its age was $t = 0.1$ s. At that time, the temperature was $T \approx 3 \times 10^{10}$ K, and the mean energy per photon was $2.7T = E_{\text{mean}} \approx 10\,\text{MeV}$. This energy is much greater than the rest energy of an electron or positron, so there were positrons as well as electrons present at $t = 0.1$ s, created by pair production:

$$\text{PAIR PRODUCTION} \quad \boxed{\gamma + \gamma \rightleftharpoons e^- + e^+.} \tag{9.8}$$

At $t = 0.1$ s, neutrinos were still coupled to the baryonic matter, and neutrons and protons could convert freely back and forth through the interactions

$$n + \nu_e \rightleftharpoons p^{+} + e^- \tag{9.9}$$

and

$$n + e^+ \rightleftharpoons p^{+} + \bar{\nu}_e. \tag{9.10}$$

At this early time, all particles, including protons and neutrons, were in kinetic equilibrium, at a temperature $kT \approx 3\,\text{MeV} \ll m_p c^2$. Thus, the number density of neutrons and protons can be found from Equation 8.26, which gives the correct number density for nonrelativistic particles in kinetic equilibrium. For neutrons,

Maxwell-Boltzmann distrib.

(8.26) for neutrons

$$n_n = g_n \left(\frac{m_n kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{m_n c^2}{kT}\right), \quad \text{for } \mu_n = 0 \tag{9.11}$$

and for protons,[3]

[2] The half-life, the time it takes for half the neutrons to decay, is related to the decay time by the relation $t_{1/2} = \tau_n \ln 2 = 610\,\text{s}$.

[3] I have left out the chemical potential terms, μ_n and μ_p, in Equations 9.11 and 9.12. At the high energies present in the early universe, as it turns out, chemical potentials are small enough to be safely neglected.

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(8.26) for protons
$$n_p = g_p \left(\frac{m_p kT}{2\pi \hbar^2} \right)^{3/2} \exp\left(-\frac{m_p c^2}{kT} \right). \quad \text{for } \mu_p = 0 \tag{9.12}$$

Since the statistical weights of protons and neutrons are equal, with $g_p = g_n = 2$, the neutron-to-proton ratio, from Equations 9.11 and 9.12, is

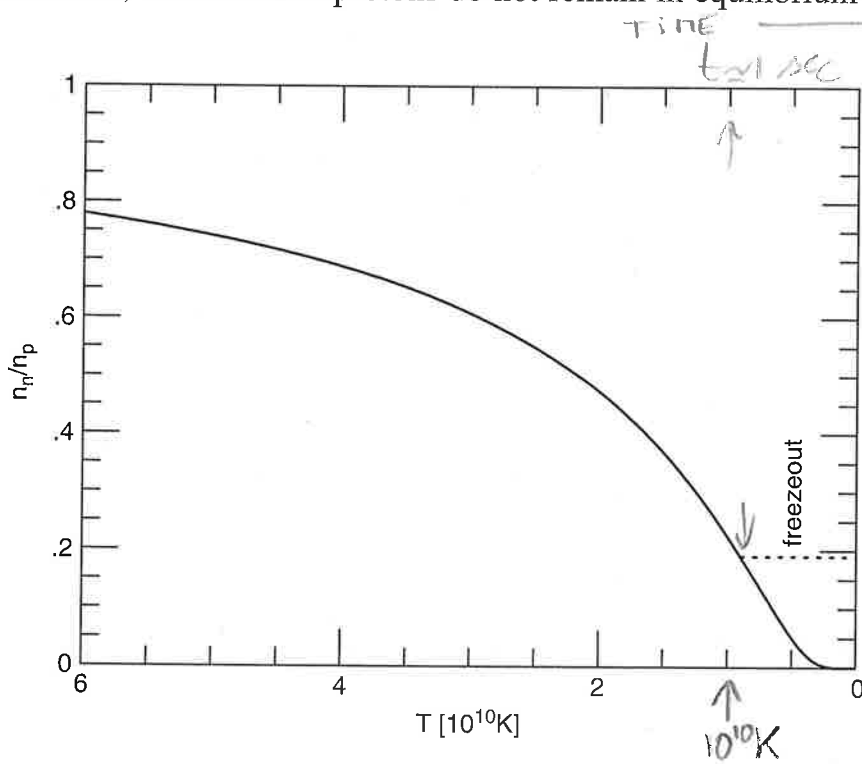
$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p} \right)^{3/2} \exp\left(-\frac{(m_n - m_p)c^2}{kT} \right). \tag{9.13}$$

The above equation can be simplified. First, $(m_n/m_p)^{3/2} = 1.002$; there will be no great loss in accuracy if we set this factor equal to one. Second, the difference in rest energy of the neutron and proton is $(m_n - m_p)c^2 = Q_n = 1.29\,\text{MeV}$. Thus, the equilibrium neutron-to-proton ratio has the particularly simple form (9.6)

NEUTRON/PROTON RATIO HISTORY
$$\frac{n_n}{n_p} = \exp\left(-\frac{Q_n}{kT} \right) \simeq \exp\left(\frac{-(m_n - m_p)c^2}{kT} \right) = \exp\left(\frac{-1.29\,\text{MeV}}{kT} \right) \tag{9.14}$$

illustrated as the solid line in Figure 9.2. At temperatures $kT \gg Q_n = 1.29\,\text{MeV}$, corresponding to $T \gg 1.5 \times 10^{10}\,\text{K}$ and $t \ll 1\,\text{s}$, the number of neutrons is nearly equal to the number of protons. However, as the temperature starts to drop below 1.5×10^{10} K, protons begin to be strongly favored, and the neutron-to-proton ratio plummets exponentially.

If the neutrons and protons remained in equilibrium, then by the time the universe was six minutes old, there would be only one neutron for every million protons. However, neutrons and protons do not remain in equilibrium for nearly



$$\frac{n_n}{n_p} \simeq \exp\left(\frac{-1.29\,\text{MeV}}{kT(t)} \right) \simeq \exp\left(\frac{-1.29\,\text{MeV}/k}{10^{10}\text{K} \cdot t_{sec}^{-1/2}} \right) \simeq \exp\left(-1.29\, t_{sec}^{1/2} \right)$$
(9.1)

Figure 9.2 Neutron-to-proton ratio in the early universe. The solid line assumes equilibrium; the dotted line gives the value after freezeout. Temperature decreases, and thus time increases, from left to right.

that long. The interactions that mediate between neutrons and protons in the early universe, shown in Equations 9.9 and 9.10, involve the interaction of a baryon with a neutrino (or antineutrino). Neutrinos interact with baryons via the weak nuclear force. At the temperatures we are considering, the cross-sections for weak interactions are small. At temperatures $kT \sim 1\,\text{MeV}$, the cross-section for the interaction of a neutrino with a proton or neutron is $T \propto t^{-1/2}$ (9.2)

$$\sigma_w \sim 10^{-47}\,\text{m}^2 \left(\frac{kT}{1\,\text{MeV}}\right)^2 \propto 10^{-47}\,\text{m}^2\, t_{sec}^{-1} \qquad (9.15)$$

$\frac{\sigma_w}{\sigma_{e.m.}} = \frac{10^{-47}\left(\frac{kT}{1\,\text{MeV}}\right)^2}{6.7\times10^{-29}} \simeq 10^{-19}\left(\frac{kT}{1\,\text{MeV}}\right)^2$ + $T \propto \frac{1}{a(t)} \propto t^{-1/2}$ $\Rightarrow \frac{\sigma_w}{\sigma_e} \simeq 10^{-19}\left(\frac{t}{1\,\text{s}}\right)^{-1}$

(Compare this to the Thomson cross-section for the interaction of electrons via the electromagnetic force: $\sigma_e = 6.65 \times 10^{-29}\,\text{m}^2$.) In the radiation-dominated universe, the temperature falls at the rate $T \propto a(t)^{-1} \propto t^{-1/2}$, and thus the cross-sections for weak interactions diminish at the rate $\sigma_w \propto t^{-1}$. The number density of neutrinos falls at the rate $n_\nu \propto a(t)^{-3} \propto t^{-3/2}$, and hence the rate Γ with which neutrons and protons interact with neutrinos via the weak force falls as a steep power of time:

see cartoon slide cylinder at (8.13) (8.14)

$$\Gamma = n_\nu c \sigma_w \propto t^{-5/2} \simeq H(t) \qquad (9.16)$$

Meanwhile, the Hubble parameter is decreasing only at the rate $H \propto t^{-1}$. When $\Gamma \approx H$, the neutrinos decouple from the neutrons and protons, and the ratio of neutrons to protons is "frozen" (at least until the neutrons start to decay, at times $t \sim \tau_n$). An exact calculation of the temperature T_{freeze} at which $\Gamma = H$ requires a knowledge of the exact cross-section of the proton and neutron for weak interactions. Using the best available laboratory information, the "freezeout temperature" turns out to be $kT_{\text{freeze}} = 0.8\,\text{MeV}$, or $T_{\text{freeze}} = 9 \times 10^9\,\text{K}$. The universe reaches this temperature when its age is $t_{\text{freeze}} \sim 1\,\text{s}$. The neutron-to-proton ratio, once the temperature drops below T_{freeze}, is frozen at the value

$kT_{freeze} \simeq 10^{10}\,\text{K} \simeq 1\,\text{MeV}$

When $T \lesssim T_{freeze}$? $\lesssim 10^{10}\,\text{K}$ $\Rightarrow$

$$\frac{n_n}{n_p} \overset{(9.14)}{=} \exp\left(-\frac{Q_n}{kT_{\text{freeze}}}\right) \approx \exp\left(-\frac{1.29\,\text{MeV}}{0.8\,\text{MeV}}\right) \approx 0.2 \simeq \frac{1}{5} \qquad (9.17)$$

At times $t_{\text{freeze}} < t \ll \tau_n$, there was one neutron for every five protons in the universe.

The scarcity of neutrons relative to protons explains why Big Bang nucleosynthesis was so incomplete, leaving three-fourths of the baryons in the form of unfused protons. A neutron will fuse with a proton much more readily than a proton will fuse with another proton. When a proton and neutron fuse to form a deuteron, the reaction is straightforward:

[COMP TO: STRONG REACTION $p^+ + n \rightleftharpoons \text{D} + \gamma. \qquad (9.18)$ ⟺ FAST!

WEAK " $p^+ + p^+ \rightarrow \text{D} + e^+ + \nu_e$ ⟺ SLOW!]

There is no Coulomb barrier between the proton and neutron, and the reaction is mediated by the strong nuclear force. Thus, it has a large cross-section and a fast reaction rate. By contrast, the fusion of two protons to form a deuteron is

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an inefficient two-step process. First, the protons must overcome the Coulomb barrier between them to form a *diproton*, otherwise known as helium-2:

$^4He \Leftrightarrow {}^2He$

WEAK REACTION $$\overset{+}{p}+\overset{+}{p} \rightleftharpoons {}^2\mathrm{He}. \quad \Leftrightarrow \text{SLOW !} \tag{9.19}$$

$2p^+ + 2n \Leftrightarrow 2p^+ \; 0n$ "Diproton"

A diproton is wildly unstable, and splits back to a pair of free protons with a lifetime $\tau_{\mathrm{split}} \sim 10^{-23}$ s. There is, however, another possible decay mode for the diproton. It can decay to a deuteron through the reaction

$$^2\mathrm{He} \rightarrow \mathrm{D} + e^+ + \nu_e, \tag{9.20}$$

which is the required second step of proton–proton fusion. The presence of a neutrino in Equation 9.20 tells us that, like the decay of a free neutron (Equation 9.7), it involves the weak nuclear force. The lifetime for decay through the reaction in Equation 9.20 has not been directly measured. However, decays that happen via the weak nuclear force all have relatively long lifetimes, with $\tau_{\mathrm{weak}} > 10^{-2}$ s. Given the huge disparity between the lifetimes of the two possible decay modes for the diproton, the probability that a diproton will decay to a deuteron rather than back to a pair of free protons is tiny: $P \approx \tau_{\mathrm{split}}/\tau_{\mathrm{weak}} < 10^{-21}$ $\simeq 10^{-23}/10^{-2}$!

It's possible, given enough time, to coax protons into fusing with each other. It's happening in the Sun, for instance, even as you read this sentence. However, fusion in the Sun is a very slow process. If you pick out any particular proton in the Sun's core, it has only one chance in ten billion of being fused into a deuteron during the next year, despite forming short-lived diprotons millions of times per second. The core of the Sun, though, is a stable environment; it's in hydrostatic equilibrium, and its temperature and density change only slowly with time. In the early universe, by strong contrast, the temperature drops as $T \propto t^{-1/2}$ and the density of baryons drops as $n_{\mathrm{bary}} \propto t^{-3/2}$. Big Bang nucleosynthesis is a race against time. After less than an hour, the temperature and density have dropped too low for fusion to occur.

$T \propto (1+z) \propto t^{-1/2}$
$n_0 \propto (1+z)^3 \propto t^{-3/2}$

Given the alacrity of neutron–proton fusion when compared to the leisurely rate of proton–proton fusion, we can state, as a lowest order approximation, that BBN proceeds until every free neutron is bonded into an atomic nucleus, with the leftover protons remaining solitary.[4] In this approximation, we can compute the maximum possible value of Y_p, the fraction of the baryon mass in the form of ^{4}He. To compute the maximum possible value of Y_p, suppose that every neutron present after the proton–neutron freezeout is incorporated into a ^{4}He nucleus.
(9.17) $\Rightarrow$ Given a neutron-to-proton ratio of $n_n/n_p = 1/5$, we can consider a representative group of 2 neutrons and 10 protons. The 2 neutrons can fuse with 2 of the protons

$$\left.\begin{matrix} N_n = 2 \\ N_{p^+} = 10 \\ \overline{\text{TOT}\; 12} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 1\;{}^4He \Leftrightarrow 4m_p \\ +\,8\times{}^1H \Leftrightarrow \underline{8m_p} \\ 12 \end{matrix}\right\} \Rightarrow \frac{m_{He}}{m_{H+He}} \simeq \frac{4}{8+4} = \frac{4}{12} = 1/3 = Y_{max}$$

[4] For the sake of completeness, note that the rate of neutron–neutron fusion is also negligibly small compared to the rate of neutron–proton fusion. A "dineutron," like a diproton, is unstable, and has an overwhelming probability of decaying back to a pair of free neutrons rather than decaying through the weak nuclear force to a deuteron.

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to form a single ^{4}He nucleus. The remaining 8 protons, though, will remain unfused. The mass fraction of ^{4}He will then be

$Y_{MAX} = \frac{2n_n}{n_p + n_n} = \frac{2f}{1+f}$ with $f = \frac{n_n}{n_p}$

$f \Rightarrow Y_{max}$

$0.2 \Rightarrow 0.333$

$0.14 \Rightarrow 0.245$

$$Y_{\max} = \frac{4}{12} = \frac{1}{3}. \tag{9.21}$$

More generally, if $\boxed{f \equiv n_n/n_p}$, with $0 \leq f \leq 1$, then the maximum possible value of Y_p is $Y_{\max} = 2f/(1+f)$.

$= \frac{2n_n}{n_p} \Big/ \left(1 + \frac{n_n}{n_p}\right) = \frac{2n_n}{n_p} \cdot \left(\frac{n_p}{n_p+n_n}\right) = \frac{2n_n}{n_p+n_n} = Y_{max}$

Planck value!

If the observed value of $Y_p = 0.24$ were greater than the predicted $Y_{\max}$, that would be a cause for worry; it might mean, for example, that we didn't really understand the process of proton–neutron freezeout. However, the fact that the observed value of Y_p is less than $Y_{\max}$ is not worrisome; various factors act to reduce the actual value of Y_p below its theoretical maximum. First, if nucleosynthesis didn't take place immediately after freezeout at $t \approx 1$ s, then the spontaneous decay of neutrons would inevitably lower the neutron-to-proton ratio, and thus reduce the amount of ^{4}He produced. Next, if some neutrons escape fusion altogether, or end in nuclei lighter than ^{4}He (such as D or ^{3}He), they will not contribute to Y_p. Finally, if nucleosynthesis goes on long enough to produce nuclei heavier than ^{4}He, that too will reduce Y_p.

In order to compute Y_p accurately, as well as the abundances of other isotopes, it will be necessary to consider the process of nuclear fusion in more detail. Fortunately, much of the statistical mechanics we will need is a rehash of what we used when studying recombination.

9.3 Deuterium Synthesis

Let's move on to the next stage of Big Bang nucleosynthesis, just after proton–neutron freezeout is complete. The time is $t \approx 2$ s. The neutron-to-proton ratio is $n_n/n_p = 0.2$. The neutrinos, which ceased to interact with electrons about the same time they stopped interacting with neutrons and protons, are now decoupled from the rest of the universe. The photons, however, are still strongly coupled to the protons and neutrons. Big Bang nucleosynthesis takes place through a series of two-body reactions, building heavier nuclei step by step. The essential first step in BBN is the fusion of a proton and a neutron to form a deuteron:

(9.18) PROTON-NEUTRON FUSION: $$p^+ + n \rightleftharpoons D^+ + \gamma. \tag{9.22}$$ $\Leftrightarrow$ FAST

When a proton and a neutron fuse, the energy released (and carried away by a gamma-ray photon) is the binding energy of a deuteron:

$$B_{\rm D} = (m_n + m_p - m_{\rm D})c^2 = 2.22\,\text{MeV}. \tag{9.23}$$

Conversely, a photon with energy $\geq B_{\rm D}$ can photodissociate a deuteron into its component proton and neutron. The reaction shown in Equation 9.22 should have

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a haunting familiarity if you've just read Chapter 8; it has the same structural form as the reaction governing the recombination of hydrogen:

Ch 8: H-RECOMBINATION ⟺ CMB

$$p^+ + e^- \rightleftharpoons H + \gamma. \tag{9.24}$$

A comparison of Equation 9.22 with Equation 9.24 shows that in each case, two particles become bound together to form a composite object, with the excess energy carried away by a photon. In the case of nucleosynthesis, a proton and neutron are bonded by the strong nuclear force to form a deuteron, with a gamma-ray photon being emitted. In the case of ~~photoionization~~, a proton and electron are bonded by the electromagnetic force to form a neutral hydrogen atom, with an ultraviolet photon being emitted. A major difference between nucleosynthesis and recombination, of course, is between the energy scales involved.[5] recombination!

Despite the difference in energy scales, many of the equations used to analyze recombination can be re-used to analyze deuterium synthesis. Around the time of recombination, for instance, photoionization was in chemical equilibrium with radiative recombination (Equation 9.24). As a consequence, the relative numbers of free protons, free electrons, and neutral hydrogen atoms are given by the Saha equation,

(8.29) SAHA EQ:

$$\boxed{\frac{n_H}{n_p n_e} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{Q}{kT}\right),} \tag{9.25}$$

Saha equation for $T > 3980$ ionization / $T < 3980$ recombination

which tells us that neutral hydrogen is favored in the limit $kT \to 0$, and that ionized hydrogen is favored in the limit $kT \to \infty$. Around the time of deuterium synthesis, neutron–proton fusion was in chemical equilibrium with photodissociation of deuterons (Equation 9.22). As a consequence, the relative numbers of free protons, free neutrons, and deuterons are given by an equation directly analogous to Equation 8.28:

(8.28) BUT FOR DEUT!

$$\frac{n_D}{n_p n_n} = \frac{g_D}{g_p g_n}\left(\frac{m_D}{m_p m_n}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{[m_p + m_n - m_D]c^2}{kT}\right). \tag{9.26}$$

From Equation 9.23, we can make the substitution $[m_p + m_n - m_D]c^2 = B_D$. The statistical weight of the deuteron is $g_D = 3$, in comparison to $g_p = g_n = 2$ for a proton or neutron. To acceptable accuracy, we may write $m_p = m_n = m_D/2$. These substitutions yield the nucleosynthetic equivalent of the Saha equation,

$\frac{g_D}{g_p g_n}\left(\frac{m_D}{m_p}\right)^{3/2}\cdot\left(\frac{1}{2}\right)^{-3/2}\cdot m_n^{-3/2} \Rightarrow$
$\simeq \frac{3}{4}\cdot 2^{3/2}\, 2^{3/2} \simeq 6$

$$\boxed{\frac{n_D}{n_p n_n} = 6\left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{B_D}{kT}\right),} \tag{9.27}$$

Saha equation for Deuterium production at $T \simeq 6\times10^8$ K ($\simeq \frac{2.22\,\text{MeV}}{13.6\,\text{eV}} \times 3780$ K)

which tells us that deuterium is favored in the limit $kT \to 0$, and that free protons and neutrons are favored in the limit $kT \to \infty$.

[5] As the makers of bombs have long known, you can release much more energy by fusing atomic nuclei than by simply shuffling electrons around.

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To define a precise temperature T_{nuc} at which the nucleosynthesis of deuterium takes place, we need to define what we mean by "the nucleosynthesis of deuterium." Just as recombination takes a finite length of time, so does nucleosynthesis. It is useful, though, to define T_{nuc} as the temperature at which $n_{\text{D}}/n_n = 1$; that is, the temperature at which half the free neutrons have been fused into deuterons. As long as Equation 9.27 holds true, the deuteron-to-neutron ratio can be written as

$$\boxed{\frac{n_{\text{D}}}{n_n} = 6n_p \left(\frac{m_n kT}{\pi \hbar^2}\right)^{-3/2} \exp\left(\frac{B_{\text{D}}}{kT}\right).} \tag{9.28}$$

We can write the deuteron-to-neutron ratio as a function of T and the baryon-to-photon ratio η if we make some simplifying assumptions. Even today, we know that $\sim 75\%$ of all the baryons in the universe are in the form of unbound protons. Before the start of deuterium synthesis, five out of six baryons (or $\sim 83\%$) were in the form of unbound protons. Thus, if we don't want to be fanatical about accuracy, we can write (8.23) $n_\gamma \approx 0.243\,(kT/\hbar c)^3$

$$n_p \approx 0.8 n_{\text{bary}} = 0.8\eta n_\gamma = 0.8\eta \left[0.2436 \left(\frac{kT}{\hbar c}\right)^3\right], \tag{9.29}$$

$(\eta^{-1} \simeq 1.6 \times 10^9)$ $\eta \approx 6 \times 10^{-10}$

using Equation 8.23 for the number density n_γ of blackbody photons. By substituting Equation 9.29 into Equation 9.28, we find that the deuteron-to-neutron ratio is a relatively simple function of temperature:

DEUTERIUM-NEUTRON RATIO:

$$\boxed{\frac{n_{\text{D}}}{n_n} \approx 6.5\eta \left(\frac{kT}{m_n c^2}\right)^{3/2} \exp\left(\frac{B_{\text{D}}}{kT}\right).} \tag{9.30}$$

This function is plotted in Figure 9.3, assuming a baryon-to-photon ratio $\eta =$ $\eta \approx$ 6.1×10^{-10}. The temperature T_{nuc} of deuterium nucleosynthesis can be found by solving the equation

$$1 \approx 6.5\eta \left(\frac{kT_{\text{nuc}}}{m_n c^2}\right)^{3/2} \exp\left(\frac{B_{\text{D}}}{kT_{\text{nuc}}}\right). \tag{9.31}$$

With $m_n c^2 = 939.6\,\text{MeV}$, $B_{\text{D}} = 2.22\,\text{MeV}$, and $\eta = 6.1 \times 10^{-10}$, the temperature of deuterium synthesis is $kT_{\text{nuc}} \approx 0.066\,\text{MeV} \approx B_{\text{D}}/34$, corresponding to $T_{\text{nuc}} \approx$ 7×10^8 K $7.6 \times 10^8\,\text{K}$. The temperature drops to this value when the age of the universe is $(2 \times 10^9$ K$)$ $t_{\text{nuc}} \approx 200\,\text{s}$ = FIRST THREE MINUTES !

Note that the time delay until the start of nucleosynthesis, $t_{\text{nuc}} \approx 200\,\text{s}$, is not negligible compared to the decay time of the neutron, $\tau_n = 880\,\text{s}$. By the time nucleosynthesis actually gets underway, neutron decay has slightly decreased the neutron-to-proton ratio from $n_n/n_p = 1/5$ to

$$\frac{n_n}{n_p} \approx \frac{\exp(-200/880)}{5 + [1 - \exp(-200/880)]} \approx \frac{0.80}{5.20} \approx 0.15. \tag{9.32}$$

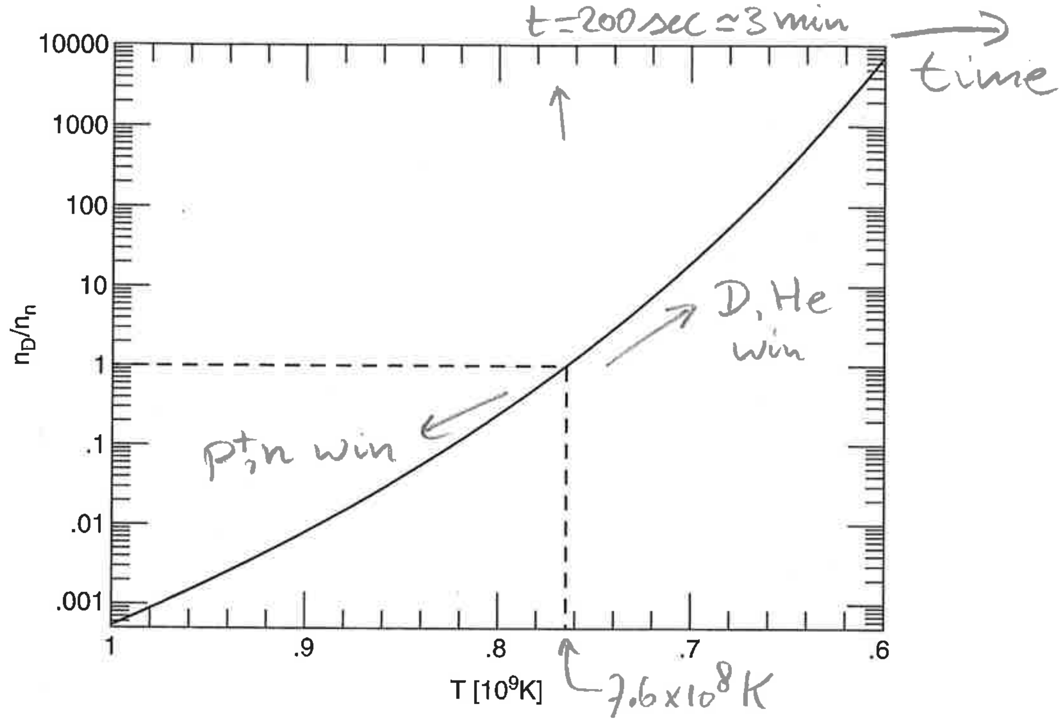

Figure 9.3 The deuteron-to-neutron ratio during the epoch of deuterium synthesis. The nucleosynthetic equivalent of the Saha equation (Equation 9.27) is assumed to hold true. Temperature decreases, and thus time increases, from left to right.

This in turn lowers the maximum possible ^{4}He mass fraction from $Y_{\max} \approx 0.33$ $= \frac{2f}{1+f}$ with $f=0.2$
to $Y_{\max} \approx$ ~~0.27~~ $0.26 = \frac{2n_n}{n_p+n_n} = \frac{2f}{1+f} = 0.261$, $f = 0.15 = \frac{n_n}{n_{p^+}}$

9.4 Beyond Deuterium

The deuteron-to-neutron ratio n_D/n_n does not remain indefinitely at the equilibrium value given by Equation 9.30. Once a significant amount of deuterium forms, many possible nuclear reactions are available. For instance, a deuteron can fuse with a proton to form ^{3}He:

$$D + p^{+} \rightleftharpoons {}^3\text{He} + \gamma. \quad \text{(3-Helium)} \tag{9.33}$$

Alternatively, it can fuse with a neutron to form ^{3}H, also known as tritium:

$$D + n \rightleftharpoons {}^3\text{H} + \gamma. \quad \text{(TRITIUM)} \tag{9.34}$$

Tritium is unstable; it spontaneously decays to ^{3}He, emitting an electron and an electron antineutrino in the process. However, the decay time of tritium is approximately 18 years; during the brief time that Big Bang nucleosynthesis lasts, tritium can be regarded as effectively stable.

Deuterons can also fuse with each other to form ^{4}He:

$$D + D \rightleftharpoons {}^4\text{He} + \gamma. \quad \text{(4-HELIUM)} \tag{9.35}$$

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However, it is more likely that the interaction of two deuterons will end in the formation of a tritium nucleus (with the emission of a proton),

either

$$\boxed{\mathrm{D}+\mathrm{D} \rightleftharpoons {}^{3}\mathrm{H}+p^{+},} \tag{9.36}$$

or the formation of a ^{3}He nucleus (with the emission of a neutron),

or

$$\boxed{\mathrm{D}+\mathrm{D} \rightleftharpoons {}^{3}\mathrm{He}+n.} \tag{9.37}$$

A large amount of ^{3}H or ^{3}He is never present during the time of nucleosynthesis. Soon after they are formed, they are converted to ^{4}He by reactions such as

V STRONG

$$\boxed{\begin{gathered} {}^{3}\mathrm{H}+p \rightleftharpoons {}^{4}\mathrm{He}+\gamma \\ {}^{3}\mathrm{He}+n \rightleftharpoons {}^{4}\mathrm{He}+\gamma \\ {}^{3}\mathrm{H}+\mathrm{D} \rightleftharpoons {}^{4}\mathrm{He}+n \\ {}^{3}\mathrm{He}+\mathrm{D} \rightleftharpoons {}^{4}\mathrm{He}+p^{+} \end{gathered}} \Longleftrightarrow \text{FAST} \tag{9.38}$$

None of the post-deuterium reactions outlined in Equations 9.33 through 9.38 involve neutrinos; they all involve the strong nuclear force, and have large cross-sections and fast reaction rates. Thus, once nucleosynthesis begins, D, ^{3}H, and ^{3}He are all efficiently converted to ^{4}He.

Once ^{4}He is reached, however, the orderly march of nucleosynthesis to heavier and heavier nuclei reaches a roadblock. For such a light nucleus, ^{4}He is exceptionally tightly bound, as illustrated in Figure 9.1. By contrast, there are no stable nuclei with $A = 5$. If you try to fuse a proton or neutron to ^{4}He, it won't work; ^{5}He and ^{5}Li are not stable nuclei. Thus, ^{4}He is resistant to fusion with protons and neutrons. Small amounts of ^{6}Li and ^{7}Li, the two stable isotopes of lithium, are made by reactions such as

$$\underset{\substack{2p\\2n}}{{}^{4}\mathrm{He}}+\underset{\substack{1p\\1n}}{\mathrm{D}} \rightleftharpoons \underset{\substack{3p\\3n}}{{}^{6}\mathrm{Li}}+\gamma \quad \text{(6-LITHIUM)} \tag{9.39}$$

and

$$\underset{\substack{2p\\2n}}{{}^{4}\mathrm{He}}+\underset{\substack{1p\\2n}}{{}^{3}\mathrm{H}} \rightleftharpoons \underset{\substack{3p\\4n}}{{}^{7}\mathrm{Li}}+\gamma. \quad \text{(7-LITHIUM)} \tag{9.40}$$

In addition, small amounts of ^{7}Be are made by reactions such as

$$\underset{\substack{2p\\2n}}{{}^{4}\mathrm{He}}+\underset{\substack{2p\\1n}}{{}^{3}\mathrm{He}} \rightleftharpoons \underset{\substack{4p\\3n}}{{}^{7}\mathrm{Be}}+\gamma. \quad \text{(7-BERYLLIUM)} \tag{9.41}$$

The synthesis of nuclei with $A > 7$ is hindered by the absence of stable nuclei with $A = 8$. For instance, if ^{8}Be is made by the reaction

$${}^{4}\mathrm{He}+{}^{4}\mathrm{He} \rightarrow {}^{8}\mathrm{Be}, \Longrightarrow 2 \times {}^{4}\mathrm{He}+\gamma \tag{9.42}$$

DECAYS FAST BACK INTO He

then the ^{8}Be nucleus falls back apart into a pair of ^{4}He nuclei with a decay time of only $\tau = 10^{-16}$ s.

The bottom line is that once deuterium begins to be formed, fusion up to the tightly bound ^{4}He nucleus proceeds very rapidly. Fusion of heavier nuclei occurs much less rapidly. The precise yields of the different isotopes involved in BBN are customarily calculated using a fairly complex computer code. The complexity is necessary because of the large number of possible reactions that can occur once deuterium has been formed, all of which have temperature-dependent cross-sections. Thus, there's a good deal of bookkeeping involved. The results of a typical BBN code, which follows the mass fraction of different isotopes as the universe expands and cools, is shown in Figure 9.4. At $t < 10\,\mathrm{s}$, when $T > 3 \times 10^9\,\mathrm{K}$, almost all the baryonic matter is in the form of free protons and free neutrons. As the deuterium density climbs upward, however, the point is eventually reached where significant amounts of ^{3}H, ^{3}He, and ^{4}He are formed. By $t \sim 1000\,\mathrm{s}$, when the temperature has dropped to $T \sim 3 \times 10^8\,\mathrm{K}$, Big Bang nucleosynthesis is essentially over. Nearly all the baryons are in the form of free protons or ^{4}He nuclei. The small residue of free neutrons decays into protons. Small amounts of D, ^{3}H, and ^{3}He are left over, a tribute to the incomplete nature of Big Bang nucleosynthesis. (^{3}H later decays to ^{3}He.) Very small amounts of ^{6}Li, ^{7}Li, and ^{7}Be are made. (^{7}Be is later converted to ^{7}Li by electron capture: $^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu_e$.)

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$t < 10\,s;\ T \geq 3\times10^9$
$\forall$ free p^+, n
$t \geq 10^3\,s;\ T \leq 3\times10^8$
$\exists$ ^{4}He, D, ^{3}He, [+ Li, Be]

The yields of D, ^{3}He, ^{4}He, ^{6}Li, and ^{7}Li depend on various physical parameters. Most importantly, they depend on the baryon-to-photon ratio η. Figure 9.5 shows the abundance of various elements produced by Big Bang nucleosynthesis, plotted as a function of η. A high baryon-to-photon ratio increases the temperature T_{nuc} at which deuterium synthesis occurs, and hence gives an earlier start to Big Bang nucleosynthesis. Since BBN is a race against the clock as the density and

(9.1) $T(t) = 10^{10}\,K \left(\frac{t}{1\,sec}\right)^{-1/2}$:

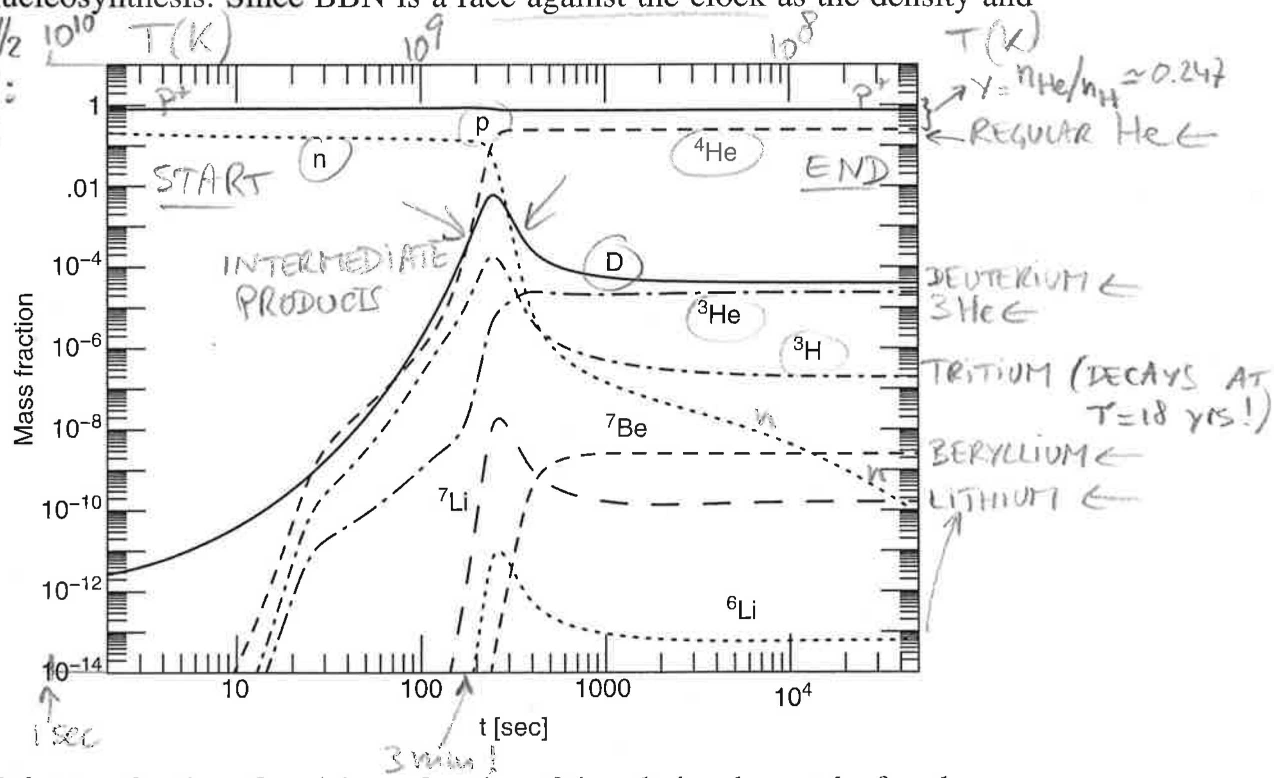

Figure 9.4 Mass fraction of nuclei as a function of time during the epoch of nucleosynthesis. Time increases, and thus temperature decreases, from left to right. [data courtesy of Alain Coc]

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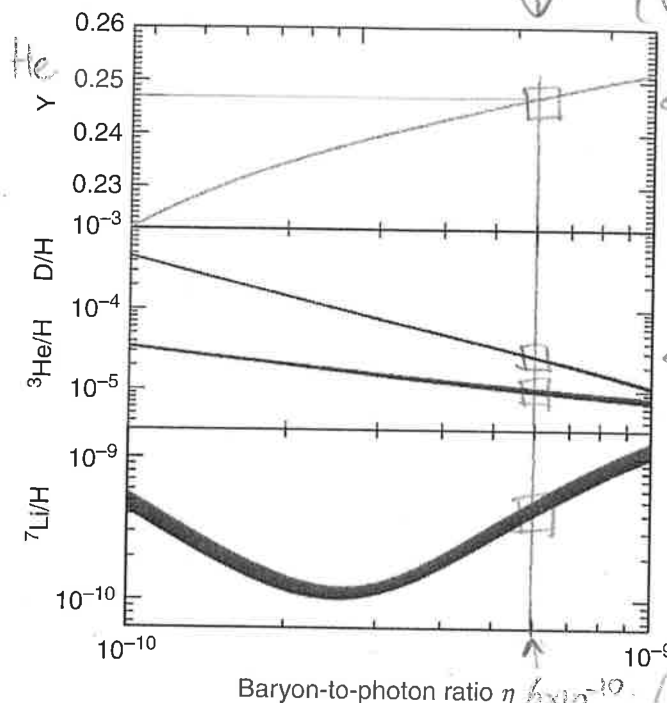
Observed in CMB & stellar spectra (POP II!)



$Y = \frac{He}{H} \simeq 0.2467 \pm .0002$ (Planck 2016)

$\frac{D}{H} \simeq 2.61 \times 10^{-5} \pm .07$ (Planck 2016)

6×10^{-10}. ($\eta^{-1} = 1.6 \times 10^{9}$)

Figure 9.5 The mass fraction of ^{4}He, and the number densities of D, ^{3}He, and ^{7}Li expressed as a fraction of the H number density. The width of each line represents the 1σ confidence interval in the density. [Cyburt *et al.* 2016, *Rev. Mod. Phys.*, **88**, 015004]

temperature of the universe drop, getting an earlier start means that nucleosynthesis is more efficient at producing ^{4}He, leaving less D and ^{3}He as leftovers. The dependence of ^{7}Li on η is more complicated. Within the range of η plotted in Figure 9.5, the direct production of ^{7}Li by the fusion of ^{4}He and ^{3}H is a decreasing function of η, while the indirect production of ^{7}Li by ^{7}Be electron capture is an increasing function of η. The net result is a minimum in the predicted density of ^{7}Li at $\eta \approx 3 \times 10^{-10}$.

To determine the value of η using the predictions of Big Bang nucleosynthesis, it is necessary to make accurate observations of the *primordial* densities of the light elements; that is, the densities before nucleosynthesis in stars started to alter the chemical composition of the universe. In determining the value of η, it is most useful to determine the primordial abundance of deuterium. This is because the deuterium abundance is strongly dependent on η in the range of interest. Thus, determining the deuterium abundance with only modest accuracy will enable us to determine η fairly well. By contrast, the primordial helium fraction, Y_p, has only a weak dependence on η for the range of interest, as shown in Figure 9.5. Thus, determining η with a fair degree of accuracy would require measuring Y_p with fanatic precision.

Deuterium abundances are customarily given as the ratio of the number of deuterium atoms to the number of ordinary hydrogen atoms (D/H). In the local interstellar gas, within $\sim 50\,\text{pc}$ of the Sun, the deuterium-to-hydrogen ratio is of

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D/H $\approx 1.6\times10^{-5}$. However, deuterium is very easily destroyed in stars. Since the interstellar gas is contaminated by deuterium-depleted gas that has been cycled through stellar interiors, we expect the primordial deuterium-to-hydrogen ratio was D/H $> 1.6 \times 10^{-5}$. Currently, the best way to find the primordial value of D/H is to look at the spectra of distant quasars. In the search for deuterium, we don't care what a quasar actually is, or how much deuterium is inside the quasar itself; instead, we just want to use the very luminous quasar as a flashlight to illuminate the intergalactic gas clouds that lie between it and us. If an intergalactic gas cloud contains no detectable stars, and has very low levels of elements heavier than lithium, we can hope that its D/H value is close to the primordial value, and hasn't been driven downward by the effects of fusion within stars. Neutral hydrogen atoms within these intergalactic clouds absorb photons whose energy corresponds to the Lyman-α transition; that is, the transition of the atom's electron from the ground state ($n = 1$) to the next higher energy level ($n = 2$). In an ordinary hydrogen atom (^{1}H), the Lyman-α transition corresponds to a wavelength $\lambda_H = 121.567$ nm (H-Lyα). In a deuterium atom, the greater mass of the nucleus causes a small isotopic shift in the electron's energy levels. As a consequence, the Lyman-α transition in deuterium corresponds to a slightly shorter wavelength, (since slightly higher energy needed for D) $\lambda_D = 121.534$ nm (D-Lyα). When we look at light from a quasar that has passed through an intergalactic cloud at redshift z_{cl}, we see a strong absorption line at $\lambda_H(1+z_{cl})$, due to absorption from ordinary hydrogen, and a much weaker absorption line at $\lambda_D(1 + z_{cl})$, due to absorption from deuterium. Detailed studies of the strength of the absorption lines in the spectra of different quasars give the ratio (D/H) $= (2.53\pm0.04)\times10^{-5}$. Using the results of BBN calculations such as those plotted in Figure 9.5, this translates into a baryon-to-photon ratio $\eta = (6.0 \pm 0.1) \times 10^{-10}$ $\Leftrightarrow \eta^{-1} \simeq 1.6\times10^{9}$, consistent with the value found from the temperature fluctuations of the cosmic microwave background (Equation 8.67).

$\Leftrightarrow$ Planck finds $\frac{D}{H} \simeq 2.61\times10^{-5} \pm .07$ (2016)

9.5 Baryon–Antibaryon Asymmetry

The results of Big Bang nucleosynthesis tell us what the universe was like when it was relatively hot ($T_{nuc} \approx 7.6 \times 10^8$ K) and dense:

$$\varepsilon_{nuc} \approx \alpha T^4_{nuc} \approx 1.6 \times 10^{33}\,\mathrm{MeV\,m^{-3}}. \tag{9.43}$$

This energy density corresponds to a mass density $\varepsilon_{nuc}/c^2 \approx 2800\,\mathrm{kg\,m^{-3}}$, or nearly three times the density of water. Remember, though, that the energy density at the time of BBN was almost entirely in the form of radiation. The mass density of baryons at the time of BBN was $T(z) = T_0(1+z) = 2.7255(1+z) \Rightarrow z_{BBN} \simeq 3\times10^{8}$!

$$\rho_{bary}(t_{nuc}) = \Omega_{bary,0}\,\rho_{c,0}\left(\frac{T_{nuc}}{T_0}\right)^3 \approx 0.009\,\mathrm{kg\,m^{-3}}. \tag{9.44}$$

A density of several grams per cubic meter is not outlandishly high, by everyday standards; it's equal to the density of the Earth's stratosphere. A mean photon energy of $2.7kT_{\rm nuc} \approx 0.18\,\text{MeV}$ is not outlandishly high, by everyday standards; you are bombarded with photons of about a third that energy when you have your teeth X-rayed at the dentist. The physics of Big Bang nucleosynthesis is well understood.

$\eta^{-1} \simeq 1.6 \times 10^{9}$

Some of the initial conditions for Big Bang nucleosynthesis, however, are rather puzzling. The baryon-to-photon ratio, $\eta \approx 6 \times 10^{-10}$, is a remarkably small number; the universe seems to have a strong preference for photons over baryons. It's also worthy of remark that the universe seems to have a strong preference for baryons over antibaryons. The laws of physics demand the presence of antiprotons ($\bar{p}$), containing two "anti-up" quarks and one "anti-down" quark apiece, as well as antineutrons ($\bar{n}$), containing one "anti-up" quark and two "anti-down" quarks apiece.[6] In practice, though, we find that the universe has an extremely large excess of protons and neutrons over antiprotons and antineutrons (and hence an excess of quarks over antiquarks). At the time of Big Bang nucleosynthesis, the number density of antibaryons ($\bar{n}$ and $\bar{p}$) was tiny compared to the number density of baryons, which in turn was tiny compared to the number density of photons. This imbalance, $n_{\rm antibary} \ll n_{\rm bary} \ll n_\gamma$, has its origin in the physics of the very early universe.

When the temperature of the early universe was greater than $kT \approx 150\,\text{MeV}$, the quarks it contained were not confined within baryons and other particles, as they are today, but formed a sea of free quarks (sometimes referred to by the oddly culinary name of "quark soup"). During the first few microseconds of the universe, when the quark soup was piping hot, quarks and antiquarks were constantly being created by pair production and destroyed by mutual annihilation:

$$\gamma + \gamma \rightleftharpoons q + \bar{q}, \tag{9.45}$$

where q and $\bar{q}$ could represent, for instance, an "up" quark and an "anti-up" quark, or a "down" quark and an "anti-down" quark. During this period of quark pair production, the numbers of "up" quarks, "anti-up" quarks, "down" quarks, "anti-down" quarks, and photons were nearly equal to each other. However, suppose there were a very tiny asymmetry between quarks and antiquarks, such that

$$\delta_q \equiv \frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \ll 1. \tag{9.46}$$

As the universe expanded and the quark soup cooled, quark–antiquark pairs would no longer be produced. The existing antiquarks would then annihilate with

[6] Note that an "anti-up" quark is *not* the same as a "down" quark; nor is "anti-down" equivalent to "up."

the quarks. However, because of the small excess of quarks over antiquarks, there would be a residue of quarks with number density

$$\frac{n_q}{n_\gamma} \sim \delta_q. \tag{9.47}$$

Thus, if there were 800 000 003 quarks for every 800 000 000 antiquarks in the early universe, three lucky quarks would be left over after the others encountered antiquarks and were annihilated. The leftover quarks, however, would be surrounded by 1.6 billion photons, the product of the annihilations. After the three quarks were bound together into a baryon at $kT \approx 150\,\text{MeV}$, the resulting baryon-to-photon ratio would be $\eta \sim 6 \times 10^{-10}$.

$\Rightarrow$ 3 quarks $\Downarrow$ 1 proton $+ 2 \times 8 \times 10^8\,\gamma$

$\Rightarrow \eta^{-1} = \frac{n_\gamma}{n_b} \sim \frac{1.6 \times 10^9}{1}$

Thus, the very strong asymmetry between baryons and antibaryons today and the large number of photons per baryon are both products of a tiny asymmetry between quarks and antiquarks in the early universe. The exact origin of the quark–antiquark asymmetry in the early universe is still not known. The physicist Andrei Sakharov, as far back as 1967, was the first to outline the necessary physical conditions for producing a small asymmetry; however, the precise mechanism by which the quarks first developed their few-parts-per-billion advantage over antiquarks still remains to be found.

Exercises

9.1 Suppose the neutron decay time were $\tau_n = 88\,\text{s}$ instead of $\tau_n = 880\,\text{s}$, with all other physical parameters unchanged. Estimate Y_{max}, the maximum possible mass fraction in ^{4}He, assuming that all available neutrons are incorporated into ^{4}He nuclei.

9.2 Suppose the difference in rest energy of the neutron and proton were $Q_n = (m_n - m_p)c^2 = 0.129\,\text{MeV}$ instead of $Q_n = 1.29\,\text{MeV}$, with all other physical parameters unchanged. Estimate Y_{max}, the maximum possible mass fraction in ^{4}He, assuming that all available neutrons are incorporated into ^{4}He nuclei.

9.3 The total luminosity of the stars in our galaxy is $L \approx 3 \times 10^{10}\,\text{L}_\odot$. Suppose that the luminosity of our galaxy has been constant for the past 10 Gyr. How much energy has our galaxy emitted in the form of starlight during that time? Most stars are powered by the fusion of H into ^{4}He, with the release of 28.4 MeV for every helium nucleus formed. How many helium nuclei have been created within stars in our galaxy over the course of the past 10 Gyr, assuming that the fusion of H into ^{4}He is the only significant energy source? If the baryonic mass of our galaxy is $M \approx 10^{11}\,\text{M}_\odot$, by what amount has the helium fraction Y of our galaxy been increased over its primordial value $Y_4 = 0.24$?