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2

Fundamental Observations

Some of the observations on which modern cosmology is based are highly complex, requiring elaborate apparatus and sophisticated data analysis. However, other observations are surprisingly simple. Let's start with an observation that is deceptive in its extreme simplicity.

2.1 The Night Sky is Dark

Step outside on a clear, moonless night, far from city lights, and look upward. You will see a dark sky, with roughly two thousand stars scattered across it. The fact that the night sky is dark at visible wavelengths, instead of being uniformly bright with starlight, is known as *Olbers' paradox*, after the astronomer Heinrich Olbers, who wrote a scientific paper on the subject in 1823. As it happens, Olbers was not the first person to think about Olbers' paradox. As early as 1576, Thomas Digges mentioned how strange it is that the night sky is dark, with only a few pinpoints of light to mark the location of stars.[1]

Why should it be paradoxical that the night sky is dark? Most of us simply take for granted the fact that daytime is bright and nighttime is dark. The darkness of the night sky certainly posed no problems to the ancient Egyptians or Greeks, to whom stars were lights stuck to a dome or sphere. However, the cosmological model of Copernicus required that the distance to stars be very much larger than an astronomical unit; otherwise, the parallax of the stars, as the Earth goes around on its orbit, would be large enough to see with the naked eye. Moreover, since the Copernican system no longer requires that the stars be attached to a rotating celestial sphere, the stars can be at different distances from the Sun. These

[1] The name "Olbers' paradox" is thus a prime example of what historians of science jokingly call the law of misonomy: nothing is ever named after the person who really discovers it. The law of misonomy is also known as "Stigler's law," after a statistician who admits that he (of course!) didn't discover it.

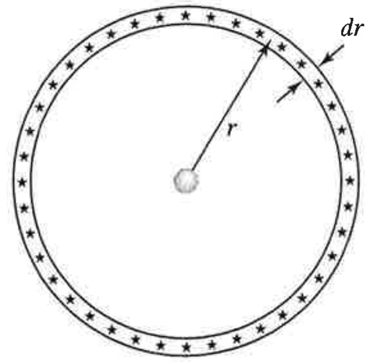

FIGURE 2.1 A star-filled spherical shell, of radius r and thickness dr, centered on the Earth.

L at a distance r is given by an inverse square law:

$$f(r) = \frac{L}{4\pi r^2}. \tag{2.1}$$

Now consider a thin spherical shell of stars, with radius r and thickness dr, centered on the Earth (Figure 2.1). The intensity of radiation from the shell of stars (that is, the power per unit area per steradian of the sky) is

$n = \rho = \frac{M}{V}$

$$dJ(r) = \frac{L}{4\pi r^2} \cdot n \cdot r^2 dr = \frac{nL}{4\pi} dr. = dJ(r) \tag{2.2}$$

The total intensity of starlight from a shell thus depends only on its thickness, not on its distance from us. We can compute the total intensity of starlight from *all* the stars in the universe by integrating over shells of all radii:

$$J = \int_{r=0}^{\infty} dJ = \frac{nL}{4\pi} \int_0^{\infty} dr = \infty. \tag{2.3}$$

Thus, I have demonstrated that the night sky is infinitely bright. $r\Big]_0^{\infty} = \infty$

This is utter nonsense.

Therefore, one (or more) of the assumptions that went into the above analysis of the sky brightness must be wrong. Let's scrutinize some of the assumptions. OLBERS ASSUMED: (1) One assumption that I made is that we have an unobstructed line of sight to every star in the universe. This is not true. In fact, since stars have a finite angular size as seen from Earth, nearby stars will hide more distant stars from our view. Nevertheless, in an infinite distribution of stars, every line of sight should end at the surface of a star; this would imply a surface brightness for the sky equal to the surface brightness of a typical star. This is an improvement on an infinitely bright sky, but is still distinctly different from the dark sky we actually see. Heinrich Olbers himself tried to resolve Olbers' Paradox by proposing that distant stars are hidden from view by interstellar matter that absorbs starlight. This resolution does not work, because the interstellar matter would be heated by starlight until it had the same temperature as the surface of a star. At that point, the interstellar

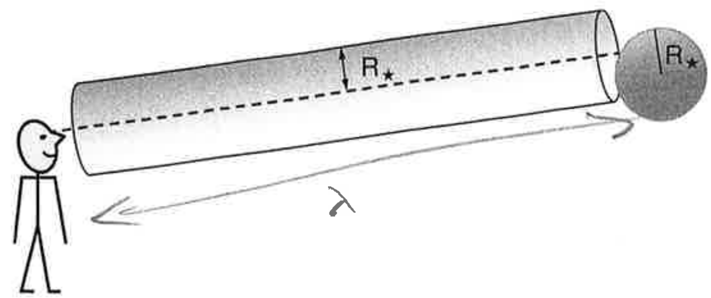

Figure 2.1 A line of sight through the universe eventually encounters an opaque star.

liberating realizations led Thomas Digges, and other post-Copernican astronomers, to embrace a model in which stars are large, opaque, glowing spheres like the Sun, scattered throughout infinite space.

$1\ pc = 3.1 \times 10^{13}$ km

$1\ Mpc = 3.1 \times 10^{22}$ m

Let's compute how bright we expect the night sky to be in an infinite universe. Let $n_\star$ be the number density of stars in the universe; averaged over large scales, this number is $n_\star \sim 10^9\,\text{Mpc}^{-3}$. Let $R_\star$ be the typical radius of a star. Although stars have a range of sizes, from dwarfs to supergiants, we may adopt the Sun as a typical mid-sized star, with $R_\star \sim R_\odot = 7.0 \times 10^8\,\text{m} = 2.3 \times 10^{-14}\,\text{Mpc}$. Consider looking outward in some direction through the universe. If you draw a cylinder of radius $R_\star$ around your line of sight, as shown in Figure 2.1, then if a star's center lies within that cylinder, the opaque star will block your view of more distant objects. If the cylinder's length is λ, then its volume is $V = \lambda\pi R_\star^2$, and the average number of stars that have their centers inside the cylinder is

$R_\odot = 7 \times 10^5$ km

$V_{cyl} = \ell \times \text{Area}$

$= \lambda \times \pi R^2$

$$N = n_\star V = n_\star \lambda \pi R_\star^2. \tag{2.1}$$

Since it requires only one star to block your view, the typical distance you will be able to see before a star blocks your line of sight is the distance λ for which $N = 1$. From Equation (2.1), this distance is

$$\lambda = \frac{1}{n_\star \pi R_\star^2}. \tag{2.2}$$

$\pi \left(\frac{7 \times 10^8\,m}{3.1 \times 10^{22}\,m}\right)^2 \simeq 1.6 \times 10^{-27}\ Mpc^2$

For concreteness, if we take $n_\star \sim 10^9\,\text{Mpc}^{-3}$ and $\pi R_\star^2 \sim \pi R_\odot^2 \sim 10^{-27}\,\text{Mpc}^2$, then you can see a distance

$$\lambda \sim \frac{1}{(10^9\,\text{Mpc}^{-3})(10^{-27}\,\text{Mpc}^2)} \sim 10^{18}\,\text{Mpc} \tag{2.3}$$

before your line of sight intercepts a star. This is a very large distance; but it is a *finite* distance. We therefore conclude that in an infinite universe (or one that stretches at least 10^{18} Mpc in all directions), the sky will be completely paved with stars.

What does this paving imply for the brightness of the sky? If a star of radius $R_\star$ is at a distance $r \gg R_\star$, its angular area, in steradians, will be

$$\Omega = \frac{\pi R_\star^2}{4\pi r^2} = \frac{R_\star^2}{4r^2}. \tag{2.4}$$

If the star's luminosity is $L_\star$, then its flux measured at a distance r will be

INVERSE SQUARE LAW
$$\boxed{f = \frac{L_\star}{4\pi r^2}.} \tag{2.5}$$

The surface brightness of the star, in watts per square meter of your pupil (or telescope mirror) per steradian, will then be

SB = CONST = r^0 EUCLIDEAN GEOMETRY
$$\Sigma_\star = \frac{f}{\Omega} = \frac{L_\star}{\pi R_\star^2}, \quad = \frac{L_\star}{4\pi r^2} \cdot \frac{1}{R_\star^2/4r^2} = \frac{L_\star}{\pi R_\star^2} \tag{2.6}$$

independent of the distance to the star. Thus, the surface brightness of a sky paved with stars will be equal to the (distance-independent) surface brightness of an individual star. We therefore conclude that in an infinite universe (or one that stretches at least 10^{18} Mpc in all directions), the entire sky, night and day, should be as dazzlingly bright as the Sun's disk. $SB(\odot) \simeq -26 - 2.5\log(\pi \cdot 900^2) \simeq -10$ mag/arcsec^2

This is utter nonsense. The surface brightness of the Sun is $\Sigma_\odot \approx 5 \times 10^{-3}$ watts m^{-2} arcsec^{-2}. By contrast, the surface brightness of the dark night sky is $\Sigma \sim 5 \times 10^{-17}$ watts m^{-2} arcsec^{-2}. Thus, my estimate of the surface brightness of the night sky ("It's the same as the Sun's") is wrong by a factor of 100 trillion.

SB(Zodi) $\simeq$ 23 mag/arcs^2

1) One (or more) of the assumptions that went into my estimate of the sky brightness must be wrong. Let's scrutinize some of the assumptions. One assumption that I made is that space is transparent over distances of 10^{18} Mpc. This might not be true. Heinrich Olbers himself tried to resolve Olbers' paradox by proposing that distant stars are hidden from view by interstellar matter that absorbs starlight. This resolution does not work in the long run, because the interstellar matter is heated by starlight until it has the same temperature as the surface of a star. At that point, the interstellar matter emits as much light as it absorbs, and glows as brightly as the stars themselves.

OLBERS ASSUMED: CAN SEE OVER 10^{18} Mpc

SB(EBL) $\simeq$ 26 mag/arcs^2

SB(DAY) $\simeq$ 1.3 mag/arcs^2

2) A second assumption that I made is that the universe is infinitely large. This might not be true. If the universe extends to a maximum distance $r_{max} \ll \lambda$, then only a fraction $F \sim r_{max}/\lambda$ of the night sky will be covered with stars. This result will also be found if the universe is infinitely large, but is devoid of stars beyond a distance r_{max}.

$R_H = \infty$ ($R_H = 4380$ Mpc)

3) A third assumption, slightly more subtle than the previous ones, is that the universe is infinitely old. This might not be true. Because the speed of light is finite, when we look farther out in space, we are looking farther out in time. Thus, we see the Sun as it was 8.3 minutes ago, Proxima Centauri as it was 4.2 years ago, and M31 as it was 2.5 million years ago. If the universe has a finite age, $t_0 \ll \lambda/c$, then we are not yet able to see stars at a distance greater than $r \sim ct_0$,

$\tau_H \simeq \frac{1}{H_0} = 13.8$ Gyr

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and only a fraction $F \sim ct_0/\lambda$ of the night sky will be covered with stars. This result will also be found if the universe is infinitely old, but has only contained stars for a finite time t_0.

4)

~~$SB \propto D^0$~~

$SB \propto (1+z)^{-4}$

$SB(z=9) \simeq 10^{-4} SB(z=0)$

A fourth assumption is that the surface brightness of a star is independent of distance, as derived in Equation 2.6. This might not be true. The assumption of constant surface brightness would have seemed totally innocuous to Olbers and other nineteenth-century astronomers, who assumed that the universe was static. However, in an expanding universe, the surface brightness of distant light sources is decreased relative to what you would see in a static universe. (In a contracting universe, the surface brightness would be increased, which would only make the problem of a bright night sky even worse.)

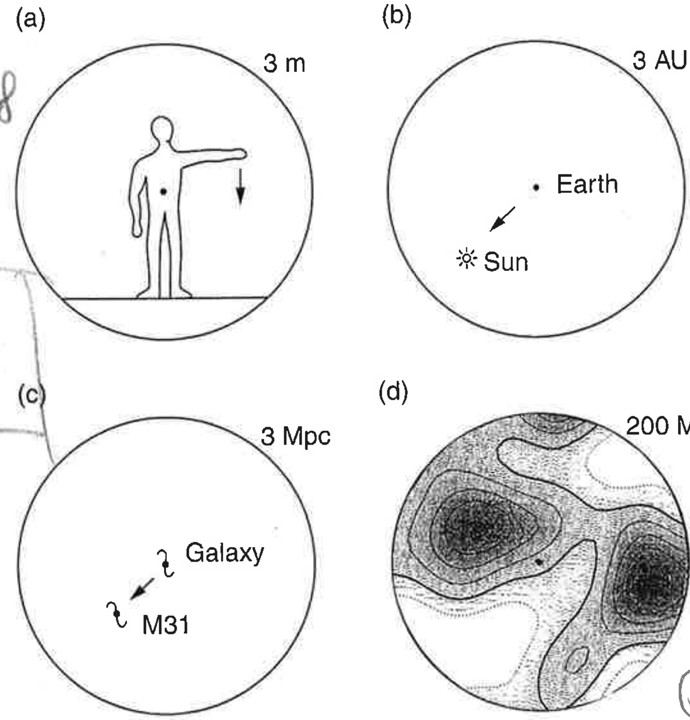
Thus, the infinitely large, eternally old, static universe that Thomas Digges and his successors pictured simply does not hold up to scrutiny. This is a textbook, not a suspense novel, so I'll tell you right now: the primary resolution to Olbers' paradox comes from the fact that the universe has a finite age. The stars beyond some finite distance, called the horizon distance, are invisible to us because their light hasn't had time to reach us yet. A particularly amusing bit of cosmological trivia is that the first person to hint at the correct resolution of Olbers' paradox was Edgar Allan Poe.[2] In his essay "Eureka: A Prose Poem," completed in 1848, Poe wrote, "Were the succession of stars endless, then the background of the sky would present us an [*sic*] uniform density... since there could be absolutely no point, in all that background, at which would not exist a star. The only mode, therefore, in which, under such a state of affairs, we could comprehend the voids which our telescopes find in innumerable directions, would be by supposing the distance of the invisible background so immense that no ray from it has yet been able to reach us at all."

2.2 The Universe is Isotropic and Homogeneous

What does it mean to state that the universe is isotropic and homogeneous? Saying that the universe is *isotropic* means that there are no preferred directions in the universe; it looks the same no matter which way you point your telescope. Saying that the universe is *homogeneous* means that there are no preferred locations in the universe; it looks the same no matter where you set up your telescope. Note the very important qualifier: the universe is isotropic and homogeneous *on large scales*. In this context, "large scales" means that the universe is only isotropic and homogeneous on scales of roughly 100 Mpc or more.

[2] That's right, the "Nevermore" guy. Poe was an excellent student at the University of Virginia (before he fell into debt and withdrew). He was then an excellent student at West Point (before he was court-martialed and expelled).

(e) $\circledast$ $\frac{\Delta\rho}{\rho}(5 M_\odot \text{ BH}) \simeq 10^{44}$ ← H/W $\boxed{2.1}$ - CONFIRM $\frac{\Delta\rho}{\rho}$ VALUES IN Fig 2.2 (a),(b),(c),(d),(e)



$\frac{\Delta\rho}{\rho} \simeq 10^{28}$

$\frac{\Delta\rho}{\rho} \simeq 10^{22}$

$\boxed{\rho \equiv \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}}$

$\frac{\Delta\rho}{\rho} \simeq 10^{1}$

$\frac{\Delta\rho}{\rho} \simeq 10^{0} = 1$

$\rho_0 = \bar{\rho} = \langle\rho\rangle \simeq 2\, m_H\, m^{-3}$

Figure 2.2 (a) A sphere 3 m in diameter, centered on your navel. (b) A sphere 3 AU in diameter, centered on your navel. (c) A sphere 3 Mpc in diameter, centered on your navel. (d) A sphere 200 Mpc in diameter, centered on your navel. Shown is the smoothed number density of galaxies. The heavy contour is drawn at the mean density; darker regions represent higher density. [Dekel *et al.* 1999, *ApJ*, **522**, 1]

$\frac{\Delta\rho}{\rho}(m) \simeq 10^{28}$ (a)

The isotropy of the universe is not immediately obvious. In fact, on small scales, the universe is blatantly anisotropic. Consider, for example, a sphere 3 m in diameter, centered on your navel (Figure 2.2a). Within this sphere, there is a preferred direction; it is the direction commonly referred to as "down." It is easy to determine the vector pointing down. Just let go of a small dense object. The object doesn't hover in midair, and it doesn't move in a random direction; it falls down, toward the center of the Earth.

$\frac{\Delta\rho}{\rho}(AU) \simeq 10^{22}$ (b)

On significantly larger scales, the universe is still anisotropic. Consider, for example, a sphere 3 AU in diameter, centered on your navel (Figure 2.2b). Within this sphere, there is a preferred direction; it is the direction pointing toward the Sun, which is by far the most massive and most luminous object within the sphere. It is easy to determine the vector pointing toward the Sun. Just step outside on a sunny day, and point to that really bright disk of light up in the sky.

$\frac{\Delta\rho}{\rho}(3\text{Mpc})$ (c) $\simeq 10^{1} = 10$

On still larger scales, the universe is *still* anisotropic. Consider, for example, a sphere 3 Mpc in diameter, centered on your navel (Figure 2.2c). This sphere contains the Local Group of galaxies, a small cluster of about a hundred galaxies. By far the most massive and most luminous galaxies in the Local Group are our own galaxy and M31, which together contribute about 86 percent of the total

M31 define a preferred direction. It is fairly easy to determine the vector pointing from our galaxy to M31. Just step outside on a clear night when the constellation Andromeda is above the horizon, and point to the fuzzy oval in the middle of the constellation.

It isn't until you get to considerably larger scales that the universe can be considered as isotropic. Consider a sphere 200 Mpc in diameter, centered on your navel. Figure 2.2d shows a slice through such a sphere, with superclusters of galaxies indicated as dark patches. The Perseus–Pisces supercluster is on the right, the Hydra–Centaurus supercluster is on the left, and the edge of the Coma supercluster is just visible at the top of Figure 2.2d. Superclusters are typically $\sim 100\,\mathrm{Mpc}$ along their longest dimensions, and are separated by voids (low density regions) which are typically $\sim 100\,\mathrm{Mpc}$ across. These are the largest structures in the universe, it seems; surveys of the universe on still larger scales don't find "superduperclusters."

$\frac{\delta\rho}{\rho}(200\,\mathrm{Mpc}) \simeq 10^0 = 1$ (d)

On small scales, the universe is obviously inhomogeneous, or lumpy, in addition to being anisotropic. For instance, a sphere 3 m in diameter, centered on your navel, will have an average density of $\sim 100\,\mathrm{kg\,m^{-3}}$, in round numbers. However, the average matter density of the universe as a whole is $\rho_0 \approx 2.7 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$. Thus, on a scale $d \sim 3\,\mathrm{m}$, the patch of the universe surrounding you is more than 28 orders of magnitude denser than average.

$m_H = 1.67\times10^{-27}$ kg

$\rho_0 = 3\times10^{-27}\ \mathrm{kg/m^3} \simeq 2\,m_H/m^3$

$\rho_0 \simeq 3\times10^{-30}\ \mathrm{gr/cm^3}$

On significantly larger scales, the universe is still inhomogeneous. A sphere 3 AU in diameter, centered on your navel, has an average density of $4 \times 10^{-5}\,\mathrm{kg\,m^{-3}}$; that's 22 orders of magnitude denser than the average for the universe.

On still larger scales, the universe is *still* inhomogeneous. A sphere 3 Mpc in diameter, centered on your navel, will have an average density of $\sim 3 \times 10^{-26}\,\mathrm{kg\,m^{-3}}$, still an order of magnitude denser than the universe as a whole. It's only when you contemplate a sphere $\sim 100\,\mathrm{Mpc}$ in diameter that a sphere centered on your navel is not overdense compared to the universe as a whole.

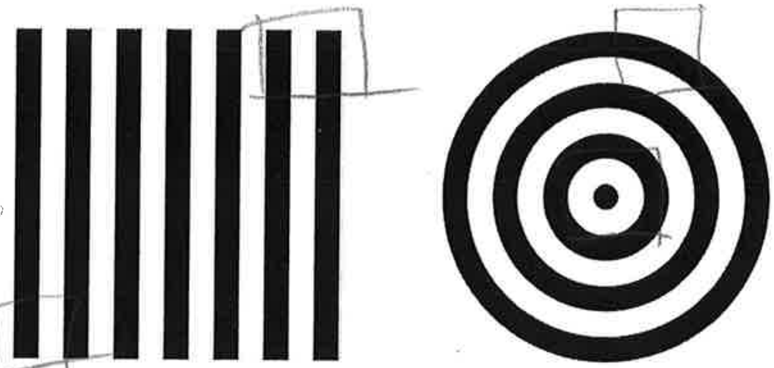
Note that homogeneity does not imply isotropy. A sheet of paper printed with stripes (Figure 2.3 left) is homogeneous on scales larger than the stripe width, but it is not isotropic. The direction of the stripes provides a preferred direction by which you can orient yourself. Note also that isotropy around a single point does not imply homogeneity. A sheet of paper printed with a bullseye (Figure 2.3 right) is isotropic around the center of the bullseye, but it is not homogeneous. The rings of the bullseye look different far from the center than they look close to the center. You can tell where you are relative to the center by measuring the radius of curvature of the nearest ring.

In general, then, saying that something is homogeneous is quite different from saying it is isotropic. However, modern cosmologists have adopted the *Copernican principle*, which states "There is nothing special or privileged about our location in the universe." The Copernican principle holds true only on large scales

(AN)ISOTROPIC – LACK OF (DIRECTIONALITY) $\Rightarrow \rho(\theta) = \text{CONST}$

(IN)HOMOGENEOUS – (NOT THE) SAME DENSITY EVERYWHERE $\Rightarrow \rho(r) = \text{CONST}$

ANISOTROPIC BUT HOMOGENEOUS



ISOTROPIC (W.R.T CENTER) BUT INHOMOGENEOUS

Figure 2.3 Left: a pattern that is anisotropic, but is homogeneous on scales larger than the stripe width. Right: a pattern that is isotropic about the origin, but is inhomogeneous.

(of 100 Mpc or more). On smaller scales, your navel obviously is in a special location. Most spheres 3 m across don't contain a sentient being; most spheres 3 AU across don't contain a star; most spheres 3 Mpc across don't contain a pair of bright galaxies. However, most spheres over 100 Mpc across do contain roughly the same pattern of superclusters and voids, statistically speaking. The universe, on scales of 100 Mpc or more, appears to be isotropic around us. Isotropy around any point in the universe, such as your navel, combined with the Copernican principle, implies isotropy around every point in the universe; and isotropy around every point in the universe *does* imply homogeneity.

The observed isotropy of the universe on scales 100 Mpc or more, combined with the assumption of the Copernican principle, leads us to state "The universe (on large scales) is homogeneous and isotropic." This statement is known as the *cosmological principle*. Although the Copernican principle forbids us to say "We're number one!", the cosmological principle permits us to say "We're second to none!"

2.3 Redshift is Proportional to Distance

When we look at a galaxy at visible wavelengths, we detect primarily the light from the stars that the galaxy contains. Thus, when we take a galaxy's spectrum at visible wavelengths, it typically contains absorption lines created in the stars' relatively cool upper atmospheres; galaxies with active galactic nuclei will also show *emission* lines from the hot gas in their nuclei. Suppose we consider a particular absorption or emission line whose wavelength, as measured in a laboratory here on Earth, is λ_{em}. The wavelength we measure for the same line in a distant galaxy's observed spectrum, λ_{ob}, will not, in general, be the same. We say that the galaxy has a redshift z, given by the formula

$$z \equiv \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}} \simeq \frac{\Delta\lambda}{\lambda}. \qquad (2.7)$$

Strictly speaking, when $z < 0$, this quantity is called a blueshift, rather than a redshift. However, the vast majority of galaxies have $z > 0$.

The fact that the light from galaxies is generally redshifted to longer wavelengths, rather than blueshifted to shorter wavelengths, was not known until the twentieth century. In 1912, Vesto Slipher at the Lowell Observatory measured the shift in wavelength of the light from the galaxy M31. He found $z = -0.001$, meaning that M31 is one of the few galaxies that exhibit a blueshift rather than a redshift. Slipher interpreted the shift in wavelength as being due to the Doppler effect. Since $|z| \ll 1$ for M31, he used the classical, nonrelativistic relation for the Doppler shift, $z = v/c$, to compute that M31 is moving toward the Earth with a speed $v = -0.001c = -300\,\mathrm{km\,s^{-1}}$.

In the year 1927, the Belgian cosmologist Georges Lemaître compiled a list of 42 galaxies whose wavelength shift had been measured, mostly by Vesto Slipher. Of these galaxies, 37 were redshifted, and only 5 were blueshifted. This is a notable excess of redshifts; by analogy, if you have a fair coin and flip it 42 times, the chance of getting "heads" 37 or more times is $P \approx 2 \times 10^{-7}$. The average radial velocity of all 42 galaxies in the sample was $v = +600\,\mathrm{km\,s^{-1}}$. Lemaître pointed out that these relatively high speeds (much higher than the average speed of stars within our galaxy) could result from an expansion of the universe. Using an estimated average distance of $r = 0.95$ Mpc for the galaxies in his sample, he concluded that the expansion was described by the parameter $K \equiv v/r = 625\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

Although Lemaître made an estimate of the average distance to the galaxies in his sample, finding an accurate distance to an individual galaxy was quite difficult. The astronomer Edwin Hubble invested a great deal of effort into measuring the distances to galaxies. By 1929, he had estimated distances for a sample of 20 galaxies whose value of z had been measured. Figure 2.4 shows Hubble's plot of redshift (z) versus distance (r) for these galaxies. He noted that the more distant galaxies had higher redshifts, and fitted the data with the famous linear relation now known as Hubble's law:

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$$\boxed{v = H_0 r} \Longleftarrow \boxed{\frac{v}{c} \simeq} z = \frac{H_0}{c} r, = \frac{r}{c/H_0} = \frac{r}{R_0} \qquad (2.8)$$

$R_0 \equiv \frac{c}{H_0} = \frac{3\times10^5}{68\ \mathrm{km/s/Mpc}} \simeq 4400$ Mpc

where H_0 is a constant (now called the Hubble constant). Interpreting the redshifts as Doppler shifts, Hubble's law takes the form

$H_0 = 68$ km/s/Mpc $\Downarrow$

$$\boxed{v = H_0 r \simeq c z} \qquad (2.9)$$

$1/H_0 =$ a Time $\simeq T_0 = t_H$

Thus, Lemaître's expansion parameter K, if we assume $z \propto r$, can be thought of as the first measurement of the Hubble constant.

The Hubble constant H_0 can be found by dividing velocity by distance, so it is customarily written in the rather baroque units of $\mathrm{km\,s^{-1}\,Mpc^{-1}}$. When Hubble first discovered Hubble's law, he thought that the numerical value of the

$v = H_0 r$

H_0 (Hubble) $= 500$ km/s/Mpc

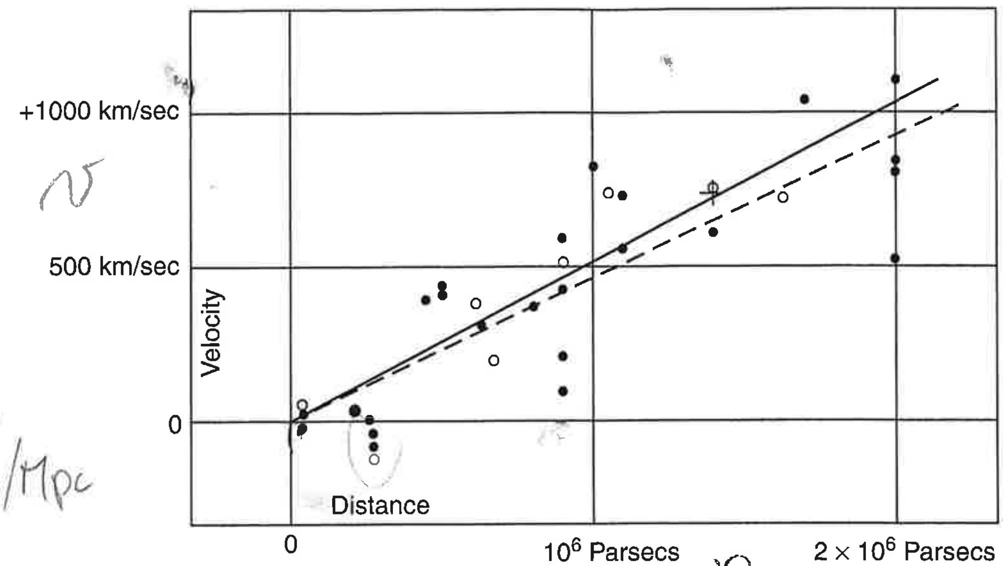
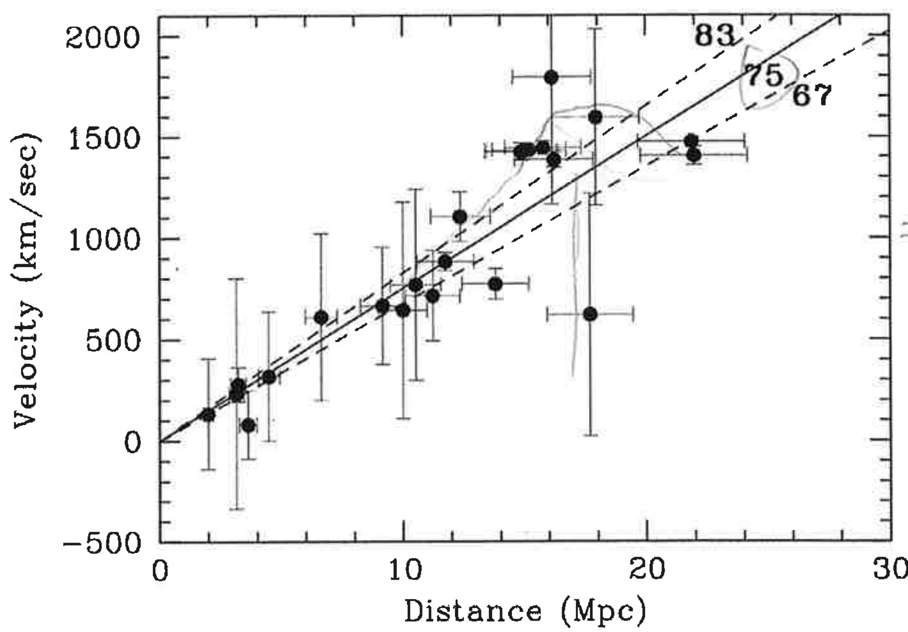


Figure 2.4 Edwin Hubble's original plot of the relation between radial velocity (assuming the formula $v = cz$) and distance. [Hubble 1929, *PNAS*, **15**, 168]

$v = H_0 r$

$H_0 = 66-68$



$N(>16) \simeq 7$

$N_{TOT} = 22$

$\Rightarrow \frac{N(>16)}{N_{TOT}} \simeq 0.32$

Figure 2.5 A more recent version of Hubble's plot, showing cz versus distance. In this case, the galaxy distances have been determined using Cepheid variable stars as standard candles, as described in Section 6.4. [Freedman *et al.* 2001, *ApJ*, **553**, 47]

Hubble constant was $H_0 \approx 500\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, as shown in Figure 2.4. However, it turned out that Hubble was severely underestimating the distances to individual galaxies, just as Lemaître, who was relying on techniques pioneered by Hubble, was underestimating the average distance to nearby galaxies.

Figure 2.5 shows a more recent determination of the Hubble constant from nearby galaxies, using data obtained using the *Hubble Space Telescope*. Notice that galaxies with a radial velocity $v = cz \approx 1000\,\mathrm{km\,s^{-1}}$, which Hubble thought

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were at a distance $r \approx 2\,\text{Mpc}$, are now more accurately placed at a distance $r \approx 15\,\text{Mpc}$. The best current estimate of the Hubble constant, combining the results from various research techniques, is

WMAP 9

$$H_0 = 68 \pm 2\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}. \quad (2.10)$$

Planck 2018 PAPERS

$H_0 \approx 66-68$

$\Delta \approx 10\%$

This is the value for the Hubble constant that we will use in the remainder of this book.

(A. RIESS) CEPHEID PAPERS $\Rightarrow H_0 \approx 71-74$

Cosmological innocents sometimes exclaim, when first encountering Hubble's law, "Surely it must be a violation of the Copernican principle to have all those distant galaxies moving away from *us*! It looks as if we are at a special location in the universe – the point away from which all other galaxies are fleeing." In fact, what we see here in our galaxy is exactly what you would expect to see in a universe that is undergoing homogeneous and isotropic expansion. We see distant galaxies moving away from us; but observers in any other galaxy would also see distant galaxies moving away from them.

YOUTUBE: "INFLATING BALLOON GALAXIES EXPAND UNIVERSE"

To see on a more mathematical level what we mean by homogeneous, isotropic expansion, consider three galaxies at positions $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}_3$. They define a triangle (Figure 2.6) with sides of length

$$r_{12} \equiv |\vec{r}_1 - \vec{r}_2| = r_{21} \quad (2.11)$$

$$r_{23} \equiv |\vec{r}_2 - \vec{r}_3| \quad (2.12)$$

$$r_{31} \equiv |\vec{r}_3 - \vec{r}_1|\,. \quad (2.13)$$

Homogeneous and uniform expansion means that the shape of the triangle is preserved as the galaxies move away from each other. Maintaining the correct relative lengths for the sides of the triangle requires an expansion law of the form

$$r_{12}(t_0) = \frac{r_{12}(t)}{a(t)} \Leftarrow$$

$$r_{12}(t) = a(t)r_{12}(t_0) \quad (2.14)$$

$$r_{23}(t) = a(t)r_{23}(t_0) \quad (2.15)$$

$$r_{31}(t) = a(t)r_{31}(t_0). \quad (2.16)$$

SCALE FACTOR

$$\boxed{a(t) = \frac{1}{1+z(t)} = \frac{r_{12}(t)}{r_{12}(t_0)} \qquad a = \frac{1}{(1+z)}}$$

$v = H_0 r$

$v(t) = H(t)\, r(t)$

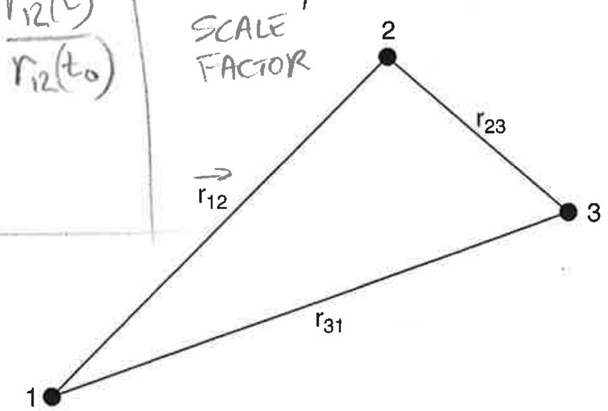

Figure 2.6 A triangle defined by three galaxies in a uniformly expanding universe.

Here the function $a(t)$ is a *scale factor*, equal to one at the present moment ($t = t_0$) and totally independent of location or direction. The scale factor $a(t)$ tells us how the expansion (or possibly contraction) of the universe depends on time. At any time t, an observer in galaxy 1 will see the other galaxies receding with a speed

$\boxed{H_0 = \frac{\dot{a}}{a}}$ $\frac{\text{km/s}}{\text{Mpc}}$

$$v_{12}(t) = \frac{dr_{12}}{dt} \underset{(2.16)}{\overset{(2.14)}{=}} \dot{a}r_{12}(t_0) = \frac{\dot{a}}{a}r_{12}(t) = H_0^{(t)} \cdot r_{12}(t) \quad (2.17)$$

$$v_{31}(t) = \frac{dr_{31}}{dt} = \dot{a}r_{31}(t_0) = \frac{\dot{a}}{a}r_{31}(t). = H_0^{(t)} \cdot r_{31}(t) \quad (2.18)$$

etc.

An observer in galaxy 2 or galaxy 3 will find the same linear relation between observed recession speed and distance, with $\dot{a}/a$ playing the role of the Hubble constant. Since this argument can be applied to any trio of galaxies, it implies that in any universe where the distribution of galaxies is undergoing homogeneous, isotropic expansion, the velocity–distance relation takes the linear form $v = Hr$, with $H = \dot{a}/a$.

If galaxies are currently moving away from each other, then it implies they were closer together in the past. Consider a pair of galaxies currently separated by a distance r, with a velocity $v = H_0 r$ relative to each other. If there are no forces acting to accelerate or decelerate their relative motion, then their velocity is constant, and the time that has elapsed since they were in contact is

$t_0 = H_0^{-1} \simeq 13.8$ Gyr

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1}, \quad \leftarrow v = \frac{r}{t} \quad (2.19)$$

(EXACT: $t_0 = 0.961\, H_0^{-1}$ – see Ry Ch 5-6)

independent of the current separation r. The time H_0^{-1} is referred to as the *Hubble time*. For $H_0 = 68 \pm 2\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$, the Hubble time is $H_0^{-1} = 14.38 \pm 0.42$ Gyr. If the relative velocities of galaxies have been constant in the past, then one Hubble time ago, all the galaxies in the universe were crammed together into a small volume. Thus, the observation of galactic redshifts led naturally to a *Big Bang* model for the evolution of the universe. A Big Bang model may be broadly defined as a model in which the universe expands from an initially highly dense state to its current low-density state.

$R_0 = \frac{c}{H_0} \simeq 4.4$ Gpc

$H_0 = 68 \frac{\text{km/s}}{\text{Mpc}}$

$H_0^{-1} = t_0$

$\boxed{\text{H/W 2.2}}$

Given H_0, calc R_0 in Gpc & t_0 in yrs exactly

The Hubble time of $\sim$ 14.4 Gyr is comparable to the ages computed for the oldest known stars in the universe. This rough equivalence is reassuring. However, the age of the universe – that is, the time elapsed since its original highly dense state – is not necessarily exactly equal to the Hubble time. We know that gravity exists, and that galaxies contain matter. If gravity working on matter is the only force at work on large scales, then the attractive force of gravity will act to slow the expansion. In this case, the universe was expanding more rapidly in the past than it is now, and the universe is somewhat younger than H_0^{-1}. On the other hand, if the energy density of the universe is dominated by a cosmological constant (an entity we'll examine in more detail in Chapter 4), then the dominant gravitational force is repulsive, and the universe may be older than H_0^{-1}.

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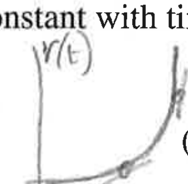
Just as the Hubble time provides a natural time scale for our expanding universe, the Hubble distance, $c/H_0 = 4380 \pm 130\,\mathrm{Mpc}$, provides a natural distance scale. The age of the universe is $t_0 \sim H_0^{-1}$, with the precise age depending on the expansion history of the universe. Even if a star began shining very early in the history of the universe, the first light from that star can only have traveled a distance $d \sim ct_0 \sim c/H_0$, with the precise travel distance depending on the expansion history of the universe. The finite age of the universe thus provides the resolution for Olbers' paradox: the night sky is dark because the light from stars at a distance much greater than c/H_0 hasn't had time to reach us.

In an infinite, eternal universe, as we have seen, you could see an average distance of $\lambda \sim 10^{18}$ Mpc before your line of sight encountered an opaque star. In a young universe where light can travel a maximum distance $d \sim c/H_0 \sim$ 4000 Mpc, the probability that you see a star along a randomly chosen line of sight is tiny: it's of order $P \sim d/\lambda \sim 4\times10^{-15}$. Thus, instead of seeing a sky completely paved with stars, with surface brightness $\Sigma \sim \Sigma_\odot \sim 5\times10^{-3}$ watts m^{-2} arcsec^{-2}, you see a sky severely underpaved with stars, with an average surface brightness[3] $\Sigma \sim P\Sigma_\odot \sim 2\times10^{-17}$ watts m^{-2} arcsec^{-2}. For the night sky to be completely paved with stars, the universe would have to be over 100 trillion times older than it is; *and* you'd have to keep the stars shining during all that time.

Hubble's law occurs naturally in a Big Bang model for the universe, in which homogeneous and isotropic expansion causes the density of the universe to decrease steadily from its initial high value. In a Big Bang model, the properties of the universe evolve with time; the average density decreases, the mean distance between galaxies increases, and so forth. However, Hubble's law can also be explained by a *Steady State* model. The Steady State model was first proposed in the 1940s by Hermann Bondi, Thomas Gold, and Fred Hoyle, who were proponents of the *perfect cosmological principle*, which states that not only are there no privileged locations in space, there are no privileged moments in time. Thus, a Steady State universe is one in which the global properties of the universe, such as the mean density ρ_0 and the Hubble constant H_0, remain constant with time.

In a Steady State universe, the velocity–distance relation

$r(t) = e^{H_0 t} = e^{t/H_0^{-1}}$



Slope $H_0 r \propto r$ AT ALL TIMES

$$\frac{dr}{dt} = H_0 r \quad = H_0 e^{H_0 t} \tag{2.20}$$

can be easily integrated, since H_0 is constant with time, to yield an exponential law:

$\exists$ NO BIG BANG!

$\Rightarrow H_0 = \frac{\dot{a}}{a} = \frac{dr/dt}{r(t)} = \frac{H_0 e^{t/H_0^{-1}}}{e^{t/H_0^{-1}}} = H_0 = \text{CONST}$

$$r(t) \propto e^{H_0 t} = e^{t/H_0^{-1}} \tag{2.21}$$

Note that $r \to 0$ only in the limit $t \to -\infty$; a Steady State universe is infinitely old. If there existed an instant in time at which the universe started expanding

[3] This crude back-of-envelope calculation doesn't exactly match the observed surface brightness of the night sky, but it's surprisingly close.

(as in a Big Bang model), that would be a special moment, in violation of the assumed "perfect cosmological principle." The volume of a spherical region of space, in a Steady State model, increases exponentially with time:

$$V = \frac{4\pi}{3} r^3 \propto e^{3H_0 t}. \tag{2.22}$$

However, if the universe is in a steady state, the density of the sphere must remain constant. To have a constant density of matter within a growing volume, matter must be continuously created at a rate

$$\dot{M}_{ss} = \rho_0 \dot{V} = \rho_0 3 H_0 V. \tag{2.23}$$

$\frac{dV}{dt} = \frac{d}{dt} e^{3H_0 t} = 3H_0 e^{3H_0 t}$

If our own universe, with matter density $\rho_0 \approx 2.7 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$, happened to be a Steady State universe, then matter would have to be created at a rate

$$\frac{\dot{M}_{ss}}{V} = 3 H_0 \rho_0 \approx 5.6 \times 10^{-28}\,\mathrm{kg\,m^{-3}\,Gyr^{-1}}. \tag{2.24}$$

CRAZY!

BULL

This corresponds to creating roughly one hydrogen atom per cubic kilometer per year.

During the 1950s and 1960s, the Big Bang and Steady State models battled for supremacy. Critics of the Steady State model pointed out that the continuous creation of matter violates mass-energy conservation. Supporters of the Steady State model pointed out that the continuous creation of matter is no more absurd than the instantaneous creation of the entire universe in a single "Big Bang" billions of years ago.[4] The Steady State model finally fell out of favor when observational evidence increasingly indicated that the perfect cosmological principle is not true. The properties of the universe *do*, in fact, change with time. The discovery of the cosmic microwave background, discussed in Section 2.5, is commonly regarded as the observation that decisively tipped the scales in favor of the Big Bang model.

2.4 Different Types of Particles

It doesn't take a brilliant observer to confirm that the universe contains a variety of different things: shoes, ships, sealing wax, cabbages, kings, galaxies, and what have you. From a cosmologist's viewpoint, though, cabbages and kings are nearly indistinguishable – the main difference between them is that the mean mass per king is greater than the mean mass per cabbage. From a cosmological viewpoint, the most significant difference between the different components of the universe is that they are made of different elementary particles. The properties of the most cosmologically important particles are summarized in Table 2.1.

[4] The name "Big Bang" was actually coined by Fred Hoyle, a supporter of the Steady State model.

Table 2.1 Elementary particle properties.

Particle	*Symbol*	*Rest energy (MeV)* mc^2	*Charge*
Proton	p	938.27	+1
Neutron	n	939.57	0
Electron	e^-	0.5110 $= m_e c^2$	−1
Neutrino	ν_e, ν_μ, ν_τ	$< 3 \times 10^{-7}$	0
Photon	γ	0	0
Dark matter	?	?	0

$\Rightarrow \frac{m_e}{m_c} \simeq 1836$

The material objects that surround us in our everyday life are made up of *protons*, *neutrons*, and *electrons*.[5] Protons and neutrons are examples of *baryons*, where a baryon is defined as a particle made of three quarks. A proton (p) contains two "up" quarks, each with an electrical charge of $+2/3$, and a "down" quark, with charge $-1/3$. A neutron (n) contains one "up" quark and two "down" quarks. Thus a proton has a net positive charge of $+1$, while a neutron is electrically neutral. Protons and neutrons also differ in their mass – or equivalently, in their rest energies. The proton mass is $m_p c^2 = 938.27\,\text{MeV}$, while the neutron mass is $m_n c^2 = 939.57\,\text{MeV}$, about 0.1% greater. Free neutrons are unstable, decaying into protons with a decay time of $\tau_n = 880\,\text{s}$, about a quarter of an hour. By contrast, protons are extremely stable; the lower limit on the decay time of the proton is $\tau_p > 10^{24} H_0^{-1}$. Neutrons can be preserved against decay by binding them into an atomic nucleus with one or more protons.

$m_p c^2 = 938$ MeV

$\Delta(m_n - m_p)c^2 \simeq 1.3$ MeV

($n \simeq 0.14\%$ HEAVIER

$\tau_p \gtrsim 10^{24} H_0^{-1} \simeq 10^{24} \cdot 13.8 \times 10^9 \cdot 31.5\,10^6 \simeq 4 \times 10^{41}$ sec $\simeq 10^{34}$ years

Electrons (e^-) are examples of *leptons*, a class of elementary particles that are not made of quarks. The mass of an electron is much smaller than that of a neutron or proton; the rest energy of an electron is $m_e c^2 = 0.511\,\text{MeV}$. An electron has an electric charge equal in magnitude to that of a proton, but opposite in sign. On large scales, the universe is electrically neutral; the number of electrons is equal to the number of protons. Since protons outmass electrons by a factor of 1836 to 1, the mass density of electrons is only a small perturbation to the mass density of protons and neutrons. For this reason, the component of the universe made up of ions, atoms, and molecules is generally referred to as *baryonic matter*, since only the baryons (protons and neutrons) contribute significantly to the mass density. Protons and neutrons are 800-pound gorillas; electrons are only 7-ounce bushbabies.

$\tau_n \simeq 14.7$ min

$m_e c^2 = 0.511$ MeV

$n_b = n_p + n_n + n_{e^-} + \dots =$ BARYON DENSITY

About three-fourths of the baryonic matter in the universe is currently in the form of ordinary hydrogen, the simplest of all elements. In addition, when we look at the remainder of the baryonic matter, it is primarily in the form of helium, the next simplest element. When astronomers look at a wide range of astronomical objects – stars and interstellar gas clouds, for instance – they find a minimum

$n_b = \frac{\ell_b}{V} = \rho_b$

[5] For that matter, we ourselves are made of protons, neutrons, and electrons.

SOLAR ABUNDANCE : $Z_\odot$: H = 70% He = 28% Me = 2% ← Metals ← BIG BANG : H: 76% He: 24% Rest: 0%

helium mass fraction of 24%. The baryonic component of the universe can be described, to lowest order, as a mix of three parts hydrogen to one part helium, with only minor contamination by heavier elements.

Another type of lepton, in addition to the electron, is the *neutrino* (ν). The most poetic summary of the properties of the neutrino was made by John Updike, in his poem "Cosmic Gall":[6]

Neutrinos, they are very small.
They have no charge and have no mass
And do not interact at all.
The earth is just a silly ball
To them, through which they simply pass,
Like dustmaids down a drafty hall
Or photons through a sheet of glass.

In truth, Updike was using a bit of poetic license here. It is definitely true that neutrinos have no electric charge. However, it is not true that neutrinos "do not interact at all"; they actually are able to interact with other particles via the weak nuclear force. The weak nuclear force, though, is very weak indeed; a typical neutrino emitted by the Sun would have to pass through a few parsecs of solid lead before having a 50 percent chance of interacting with a lead atom. Since neutrinos pass through neutrino detectors with the same facility with which they pass through the Earth, detecting neutrinos from astronomical sources is difficult.

There are three types, or "flavors," of neutrinos: electron neutrinos (ν_e), muon neutrinos (ν_μ), and tau neutrinos (ν_τ). What Updike didn't know in 1960, when he wrote his poem, is that each flavor of neutrino has a small mass. In addition to there being three flavor states of neutrino, (ν_e, ν_μ, ν_τ), there are also three *mass* states of neutrino, (ν_1, ν_2, ν_3), with masses m_1, m_2, and m_3. Each of the three flavor states is a quantum superposition of the three different mass states. The presence of three neutrino mass states, at least two of which have a non-zero mass, is known indirectly from the search for neutrino oscillations. An *oscillation* is the transmutation of one flavor of neutrino into another. For instance, an electron neutrino produced by a fusion reaction in the core of the Sun will be converted into some combination of an electron neutrino, a muon neutrino, and a tau neutrino as it moves away from the Sun. These oscillations can only occur, according to the laws of quantum mechanics, if the different mass states have masses that differ from each other. The oscillations of electron neutrinos from the Sun are explained if the two first mass states have $(m_2^2 - m_1^2)c^4 \approx 7.5 \times 10^{-5}\,\mathrm{eV}^2$. The oscillations of muon neutrinos created by cosmic rays striking the Earth's

$\Delta(m_\nu \cdot c^2)^2 = 7.5 \cdot 10^{-5}\,eV^2 \Rightarrow \sqrt{\Delta_{21}(m_\nu c^2)^2} \simeq 0.0087\,eV$

6 From TELEPHONE POLES AND OTHER POEMS, by John Updike, ©1958, 1959, 1960, 1961, 1962, 1963 by John Updike. Used by permission of Alfred A. Knopf, an imprint of the Knopf Doubleday Publishing Group, a division of Penguin Random House LLC. All rights reserved.

EXTRAGALACTIC BACKGROUND SPECTRUM

 FROM LONGAIR'S BOOK

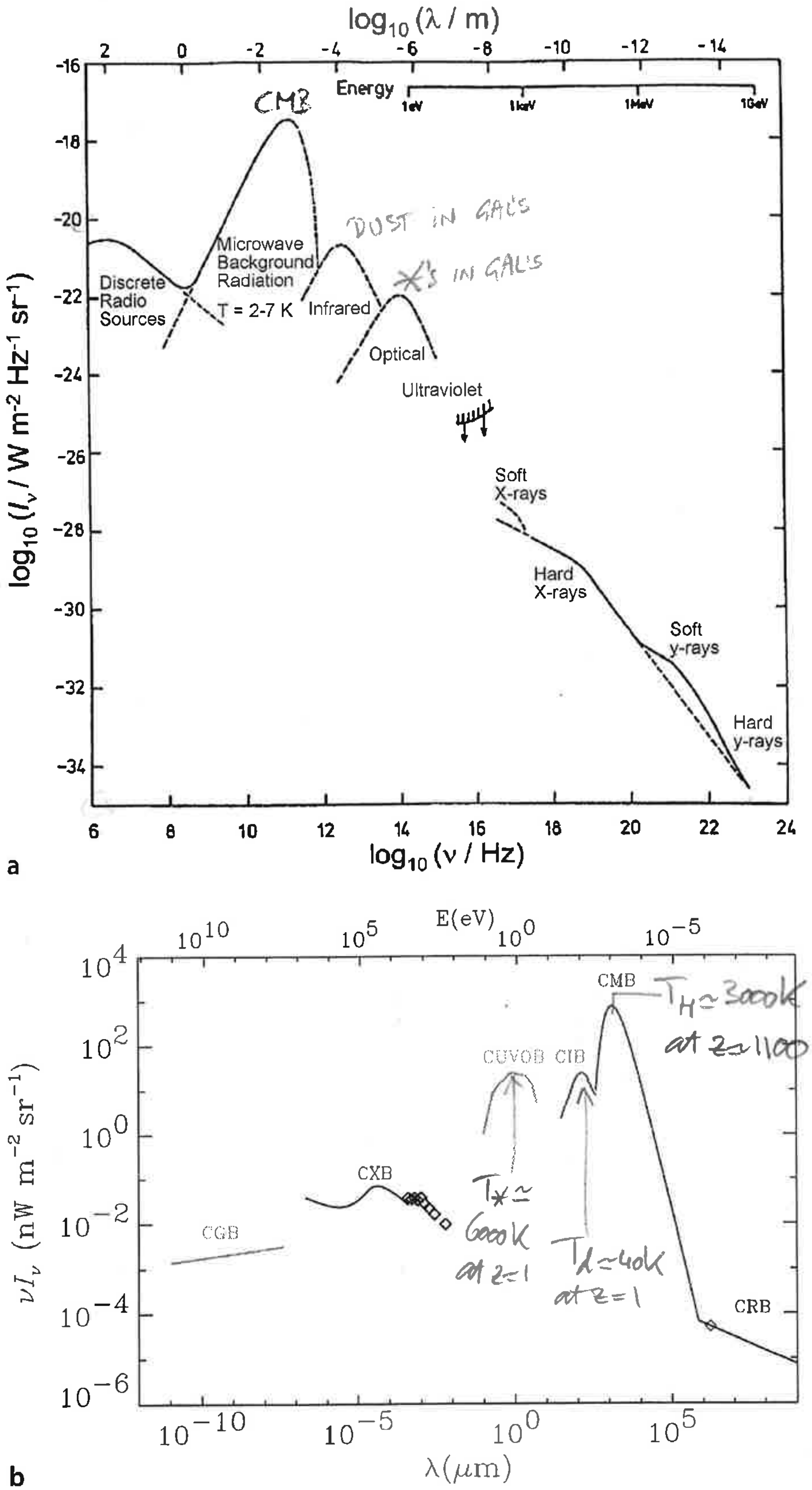
Slope: $\frac{\Delta \log I_\nu}{\Delta \log \nu}$

$\simeq \frac{-20-(-35)}{23-6} \simeq \frac{15}{17}$

$\simeq 0.9$

WHY?

$E = h\nu$



See also Windhorst et al 2018, astro-ph/1801.03584 (ApJS 234, 14)

From Longair's Book : Galaxy Formation

was dedicated to studies of the background radiation, not only in the millimetre and submillimetre wavebands, but also throughout the infrared waveband from 2 to 1000 μm.

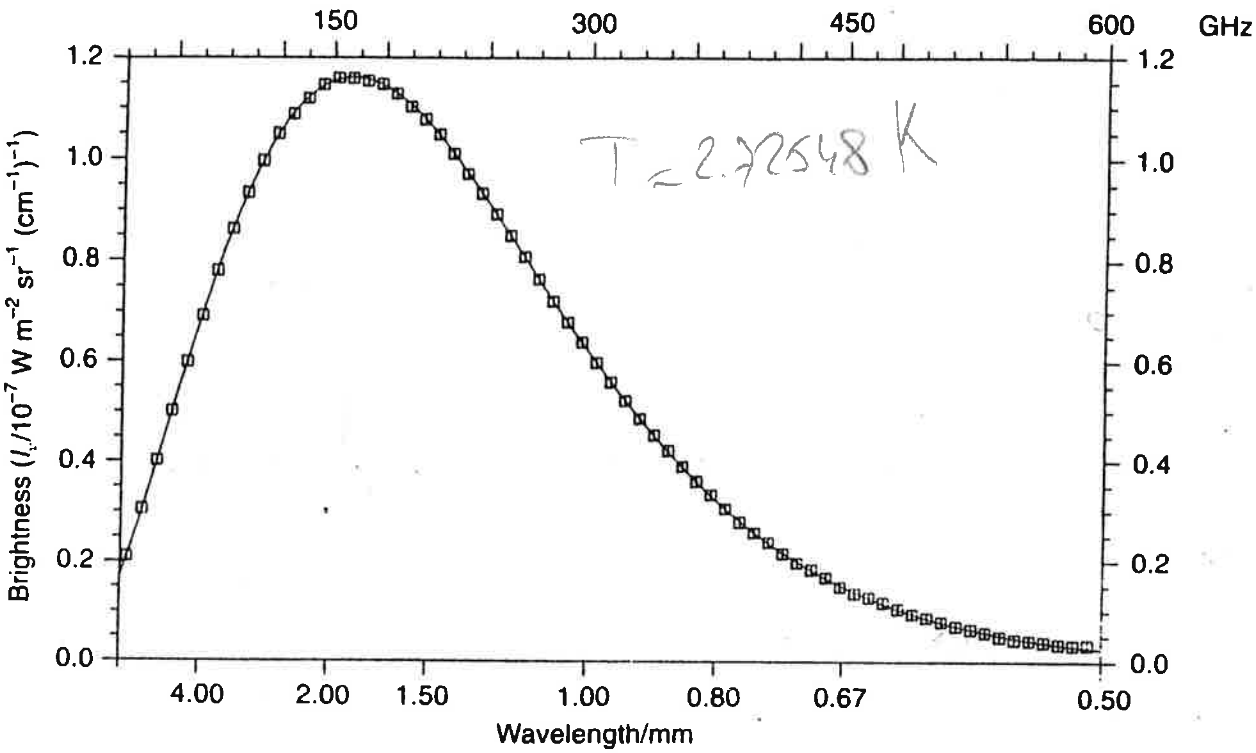


Fig. 2.1. The first published spectrum of the Cosmic Microwave Background Radiation as measured by the COBE satellite in the direction of the North Galactic Pole (Mather *et al.* 1990). Within the quoted errors, the spectrum is precisely that of a perfect black body at radiation temperature 2.735 ± 0.06 K. The more recent spectral measurements are discussed in the text. The units adopted for frequency on the ordinate are cm^{-1}. A useful conversion to more familiar units is 10^{-7} W m^{-2} sr^{-1} (cm^{-1})$^{-1}$ = 3.34×10^{-18} W m^{-2} Hz^{-1} sr^{-1} = 334 MJ sr^{-1}.

2.1.1 The Spectrum of the Cosmic Microwave Background Radiation

The Far Infrared Absolute Spectrophotometer (FIRAS) measured the *spectrum* of the Cosmic Microwave Background Radiation in the wavelength range 0.5 to 2.5 mm with very high precision during the first year of the mission. The FIRAS detectors and a reference black-body source were cooled to liquid helium temperatures and there was only sufficient liquid cryogen for one year of observation. The first observations made by FIRAS revealed that the spectrum of the background is very precisely of black-body form. In the early spectrum shown in Fig. 2.1, it can be seen that the error boxes, which are

Rayleigh-Jeans: $\nu \cdot L_\nu \propto \nu \cdot \nu^2 \propto \nu^3$

John D. Kraus (1966, McGraw Hill)

To facilitate integration let us put (Richtmyer and Kennard, 1942)

$$x = \frac{h\nu}{kT} \tag{3-54}$$

from which it follows that

$$\nu = \frac{kT}{h} x \qquad \text{and} \qquad d\nu = \frac{kT}{h} dx \tag{3-55}$$

Substituting these values into (3-53) gives

$$B' = \frac{2h}{c^2}\left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1}\, dx \tag{3-56}$$

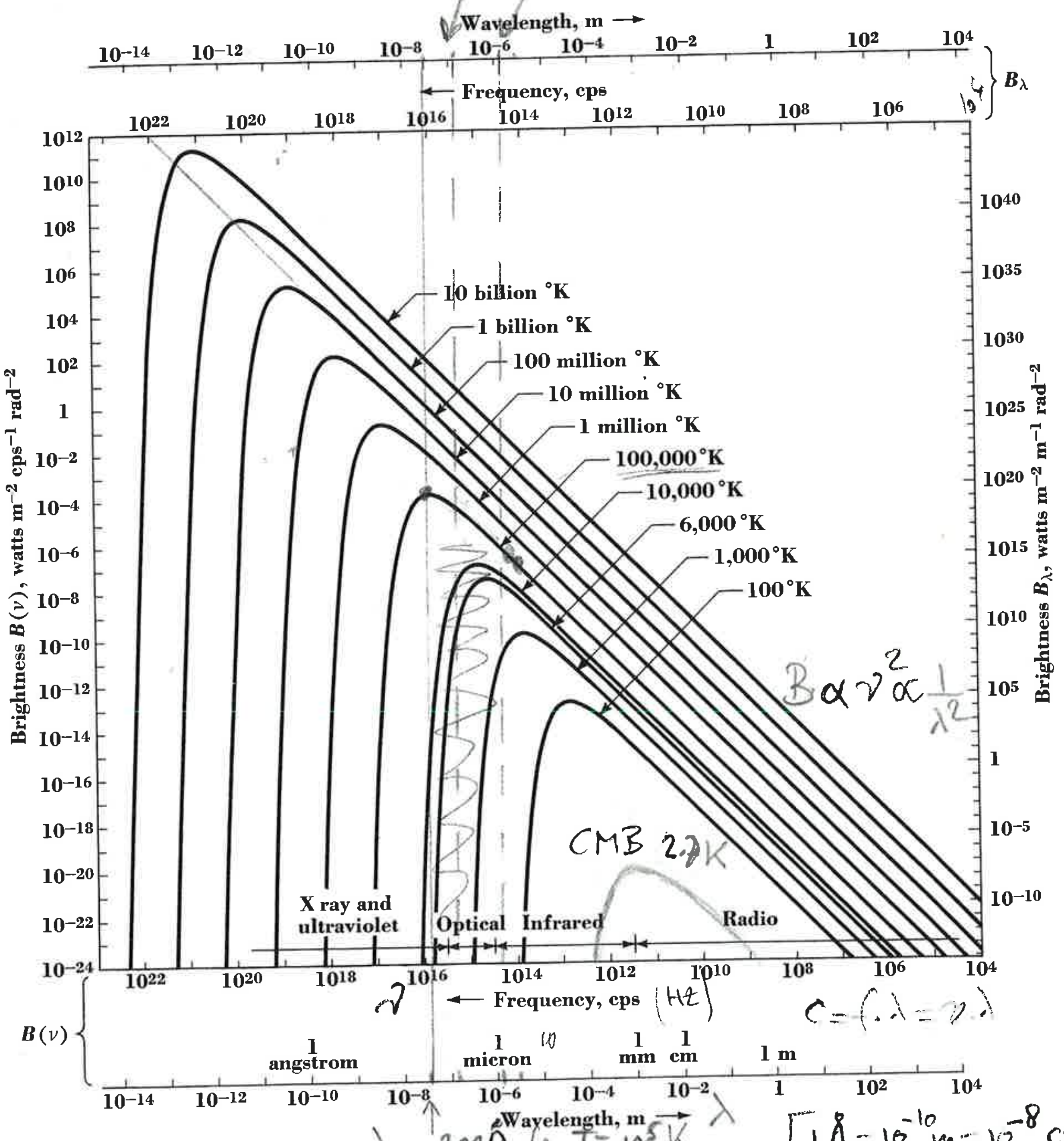


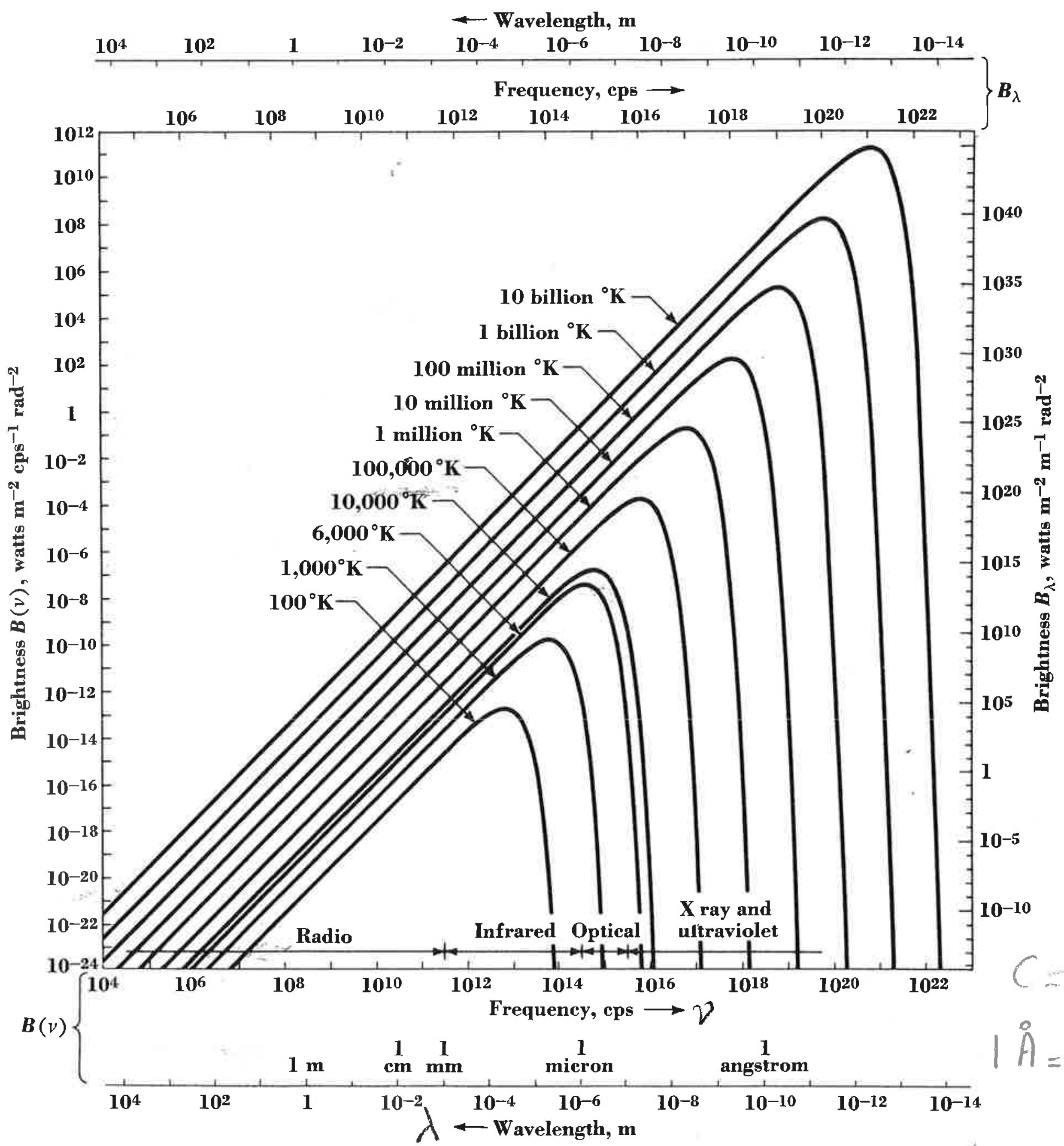
Fig. 3-13 Planck-law radiation curves to logarithmic scales with brightness expressed

The integral is a constant. Combining this constant with the others in the equation, we obtain the Stefan-Boltzmann relation*

$$B' = \sigma T^4 \tag{3-57}$$

where B' = total brightness, watts m^{-2} rad^{-2}
σ = constant ($= 1.80 \times 10^{-8}$ watt m^{-2} $°K^{-4}$)
T = temperature of blackbody, °K

The total brightness B' at a given temperature is given by the area under the Planck-radiation-law curve for that temperature. According to



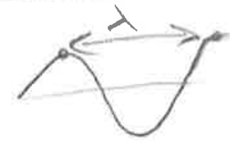
$C = f \cdot \lambda = \nu \cdot \lambda$

$1\,\text{Å} = 10^{-10}\,m = 10^{-8}\,cm$

21

upper atmosphere are most easily explained if $(m_3^2 - m_2^2)c^4 \approx 2.4 \times 10^{-3}\,\mathrm{eV}^2$. $\Rightarrow \sqrt{\Delta_{32}(m_\nu c^2)^2}$
Unfortunately, knowing the differences of the squares of the masses doesn't tell us the values of the masses themselves. Given $m_1 \geq 0$, there is a lower limit on the sum of the neutrino masses: $\simeq 0.05$ eV

$$(m_1 + m_2 + m_3)c^2 = [m(\nu_e) + m(\nu_\mu) + m(\nu_\tau)]c^2 \geq 0.057\,\mathrm{eV}. \qquad (2.25)$$



The best upper limit on the sum of the neutrino masses is given by observations of the large scale structure of the universe, as we shall see in Chapter 11. This upper limit is

Freq ≠ Vel

$\nu \equiv f$ (or: neutrino) $\nu \neq v$

$$(m_1 + m_2 + m_3)c^2 = [m(\nu_e) + m(\nu_\mu) + m(\nu_\tau)]c^2 \leq 0.3\,\mathrm{eV}\,. \qquad (2.26)$$

$\lambda . f = c = \lambda . \nu$

In any case, although John Updike was not strictly correct about neutrinos being massless, they are constrained to be very much lower in mass than electrons. $f \equiv \nu = c/\lambda$

$\lambda = \frac{c}{\nu} = \frac{c}{f}$

A particle which is known to be massless is the *photon*. Electromagnetic radiation can be thought of either as a wave or as a stream of particles, called photons. Light, when regarded as a wave, is characterized by its frequency f or its wavelength $\lambda = c/f$. When light is regarded as a stream of photons, each photon is characterized by its energy, $E_\gamma = hf$, where $h = 2\pi\hbar$ is the Planck constant. Photons of a wide range of energy, from radio to gamma rays, pervade the universe. Unlike neutrinos, photons interact readily with electrons, protons, and neutrons. For instance, photons can ionize an atom by kicking an electron out of its orbit, a process known as *photoionization*. Higher-energy photons can break an atomic nucleus apart, a process known as *photodissociation*.

$E = h\nu = \frac{hc}{\lambda}$

$\hbar = h/2\pi$

$h = 6.62\,10^{-34}$ Js

Photons, in general, are easily created. One way to make photons is to take a dense, opaque object – such as the filament of an incandescent lightbulb – and heat it up. If an object is opaque, then the protons, neutrons, electrons, and photons that it contains frequently interact, and attain thermal equilibrium; that is, they all have the same temperature T. The density of photons in the object, as a function of photon energy, will depend only on T. It doesn't matter whether the system is a tungsten filament, an ingot of steel, or a sphere of ionized hydrogen and helium. The energy density of photons in the frequency range $f \to f + df$ is given by the *blackbody* function

(1) PLANCK'S LAW: $$\varepsilon(f)df = \frac{8\pi h}{c^3}\frac{f^3\,df}{\exp(hf/kT) - 1}, \qquad (2.27)$$

illustrated in Figure 2.7.

The peak in the blackbody function occurs at $hf_{\text{peak}} \approx 2.82kT$. Integrated over all frequencies, Equation 2.27 yields a total energy density for blackbody radiation of

TOTAL ENERGY IN 2.27 Boltzmann's Constant $K = 1.38 \times 10^{-23}\,\frac{J}{K}$

(2) STEPHAN-BOLTZMANN LAW: $$\boxed{\varepsilon_\gamma = \alpha T^4,} \qquad (2.28)$$

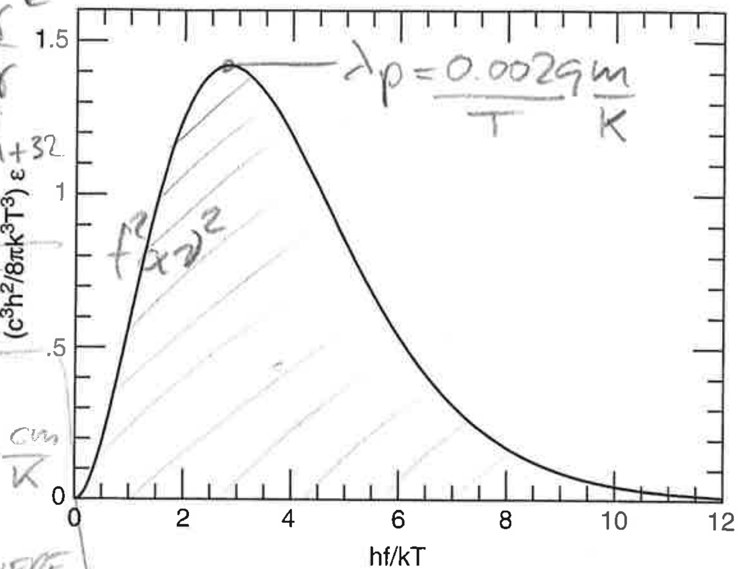
(3) WIEN'S LAW: $h \cdot \nu_p = \frac{h.c}{\lambda_p} \simeq 4.97\,KT \Rightarrow \lambda_p = \frac{hc}{4.97\,KT} \simeq \frac{0.29}{T}\left(\frac{cm}{K}\right)$

$E_{mean} = h\nu_{mean} = \frac{\varepsilon_\gamma}{n_\gamma}$ ← 2.28+2.29, ↑ 2.31+32

$E_m = \frac{\varepsilon_\gamma}{n_\gamma} = \frac{\left(\frac{\pi^2 k^4}{15(\hbar c)^3}\right) \cdot T^4}{\left(\frac{2.4 k^3}{\pi^2(\hbar c)^3}\right) \cdot T^3}$

$\Rightarrow E_{mean} \simeq \frac{\pi^4}{36} kT \simeq 2.7\,kT$

H/W 2.3 VERIFY
1) WIEN'S LAW: $\lambda_p = \frac{0.29\ cm}{T\ K}$
2) $E_{mean} \simeq 2.7kT$
FROM WHAT IS GIVEN HERE



$E = h\nu$

Figure 2.7 The energy density of blackbody radiation, expressed as a function of frequency f.

For H/W 2.3 NEED S-B LAW: $\varepsilon_\gamma = \frac{c}{4} \cdot \alpha T^4 \equiv \sigma \cdot T^4$ with $\sigma = 5.67 \times 10^{-8}\ W\,m^{-2}\,K^{-4}$

where

STEPHAN BOLTZMANN'S CONSTANT

$$\alpha = \frac{\pi^2}{15} \frac{k^4}{\hbar^3 c^3} = 7.566 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}. \tag{2.29}$$

with $\boxed{\varepsilon_\gamma = \alpha T^4}$

Since the energy of a photon is $\boxed{E_\gamma = hf}$ ($E = h\nu$), the number density of photons in the frequency range $f \rightarrow f + df$ is, from Equation 2.27,

$$n(f)df = \frac{\varepsilon(f)df}{hf} = \frac{8\pi}{c^3} \frac{f^2\,df}{\exp(hf/kT) - 1}. \tag{2.30}$$

Integrated over all frequencies, the number density of photons in blackbody radiation is

$$\boxed{n_\gamma = \beta T^3,} \tag{2.31}$$

where

$$\beta = \frac{2.4041}{\pi^2} \frac{k^3}{\hbar^3 c^3} = 2.029 \times 10^7\,\mathrm{m^{-3}\,K^{-3}}. \tag{2.32}$$

Division of Equation 2.28 by Equation 2.31 yields a mean photon energy $E_{\rm mean} = hf_{\rm mean} \approx 2.70kT$, close to the peak in the spectrum. You have a temperature $T_{\rm you} = 310\,\mathrm{K}$, assuming you are not running a fever, and you radiate an approximate blackbody spectrum, with a mean photon energy $E_{\rm mean} \approx 0.072\,\mathrm{eV}$, corresponding to a wavelength $\lambda \approx 1.7 \times 10^{-5}\,\mathrm{m} \approx 17\,000\,\mathrm{nm}$, in the mid-infrared. By contrast, the Sun produces an approximate blackbody spectrum with a temperature $T_\odot \approx 5800\,\mathrm{K}$. This implies a mean photon energy $E_{\rm mean} \approx 1.3\,\mathrm{eV}$, corresponding to $\lambda \approx 9.0 \times 10^{-7}\,\mathrm{m} \approx 900\,\mathrm{nm}$, in the near infrared. Note, however, that although the mean photon energy in a blackbody spectrum is $2.70kT$, Figure 2.7 shows us that there is a long exponential tail to higher photon energies. A large

$\lambda_{you} \simeq \frac{0.29}{310} \simeq 17\,\mu m$

$\lambda_p(\odot) = \frac{0.29}{5772\,K} = 5 \times 10^{-7}\,m$

fraction of the Sun's output is at wavelengths of $400 \rightarrow 700$ nm, which our eyes are equipped to detect.

A more mysterious component of the universe is *dark matter*. When observational astronomers refer to dark matter, they often mean any massive component of the universe that is too dim to be detected readily using current technology. Theoretical astrophysicists often use a more stringent definition of dark matter than do observers, defining dark matter as any massive component of the universe which doesn't emit, absorb, or scatter light at all.[7] If neutrinos have mass, for instance, as the neutrino oscillation results indicate, they qualify as dark matter. In some extensions to the Standard Model of particle physics, there exist massive particles that interact, like neutrinos, only through the weak nuclear force and through gravity. These particles, which have not yet been detected in the laboratory, are generically referred to as weakly interacting massive particles, or WIMPs.

In this book, we will generally adopt the broader definition of dark matter as something which is too dim for us to see, even with our best available technology. Detecting dark matter is, naturally, difficult. The standard method of detecting dark matter is by measuring its gravitational effect on luminous matter, just as the planet Neptune was first detected by its gravitational effect on the planet Uranus. Although Neptune no longer qualifies as dark matter, observations of the motions of stars within galaxies and of galaxies within clusters indicate that a significant amount of dark matter is in the universe. Exactly how much there is, and what it's made of, is a topic of great interest to cosmologists.

2.5 Cosmic Microwave Background

The discovery of the cosmic microwave background (CMB) by Arno Penzias and Robert Wilson in 1965 has entered cosmological folklore. Using a microwave antenna at Bell Labs, they found an isotropic background of microwave radiation. More recently, space-based experiments have revealed that the cosmic microwave background is exquisitely well fitted by a blackbody spectrum (Equation 2.27) with a temperature

$$\boxed{T_0 = 2.7255 \pm 0.0006\,\mathrm{K}.} \Rightarrow \lambda_p(\mathrm{CMB}) = \frac{0.0029}{2.7255} = 0.0011\,\mathrm{m} \simeq 1.1\,\mathrm{mm} \qquad (2.33)$$

The energy density of the CMB is, from Equation 2.28,

$$\underline{\varepsilon_\gamma = 4.175 \times 10^{-14}\,\mathrm{J\,m^{-3}}} = \underline{0.2606\,\mathrm{MeV\,m^{-3}}}. \qquad (2.34)$$

for 2.725 K

[7] Using this definition, an alternate name for dark matter might be "transparent matter" or "invisible matter." However, the name "dark matter" is the commonly adopted term.

$\eta^{-1} = \frac{n_\gamma}{n_H} \simeq \frac{4\times 10^8\ m^{-3}}{2\ m^{-3}} \simeq 2\times 10^8$ = PHOTON TO BARYON RATIO ⇒ LENGTH OF He-PRODUCTION

$n_H = 2\, m_H \cdot m^{-3}$ SEE CHAPTER 9.4!

$\boxed{Y = \frac{He}{H} = 0.24}$

BB PRODUCES EXACTLY 24% He (Ry Ch 9!)

The number density of CMB photons is, from Equation 2.31,

$$\boxed{n_\gamma = 4.107 \times 10^8\,\mathrm{m}^{-3}.} \tag{2.35}$$

Thus, there are about 411 CMB photons per cubic centimeter of the universe at the present day. The mean energy of CMB photons, however, is quite low, only

$$\boxed{E_{\mathrm{mean}} = 6.344 \times 10^{-4}\,\mathrm{eV}.} \quad = 2.7\,kT \tag{2.36}$$

This is too low in energy to photoionize an atom, much less photodissociate a nucleus. The mean CMB photon energy corresponds to a wavelength of 2 millimeters, in the microwave region of the electromagnetic spectrum – hence the name "cosmic *microwave* background."

The existence of the CMB is a very important cosmological clue. In particular, it is the clue that caused the Big Bang model for the universe to be favored over the Steady State model. In a Steady State universe, the existence of blackbody radiation at 2.7255 K is not easily explained. In a Big Bang universe, however, a cosmic background radiation arises naturally if the universe was initially very hot as well as very dense. If mass is conserved in an expanding universe, then in the past the universe was denser than it is now. Assume that the early dense universe was very hot ($T \gg 10^4$ K, or $kT \gg 1$ eV). At such high temperatures, the baryonic matter in the universe was completely ionized, and the free electrons rendered the universe opaque. A dense, hot, opaque body, as described in Section 2.4, produces blackbody radiation. So, the early hot dense universe was full of photons, banging off the electrons like balls in a pinball machine, with a spectrum typical of a blackbody (Equation 2.27). However, as the universe expanded, it cooled. Eventually, the temperature became sufficiently low that ions and electrons combined to form neutral atoms. When the universe no longer contained a significant number of free electrons, the blackbody photons started streaming freely through the universe, without further scattering off free electrons.

cosmic $T(z)$

$T(z=0) = 2.73\,K = T_0$

$T(z) = T_0 \cdot (1+z)$

$T(z=1090) = 3000\,K$ WHEN $p^+ + e^- \Rightarrow H$

The blackbody radiation that fills the universe today can be explained as a relic of the time when the universe was sufficiently hot and dense to be opaque. However, at the time the universe became transparent, its temperature was 2970 K. The temperature of the CMB today is 2.7255 K, a factor of 1090 lower. The drop in temperature of the blackbody radiation is a direct consequence of the expansion of the universe. Consider a region of volume V that expands at the same rate as the universe, so that $V \propto a(t)^3$. The blackbody radiation in the volume can be thought of as a photon gas with energy density $\boxed{\varepsilon_\gamma = \alpha T^4.}$ Moreover, since the photons in the volume have momentum as well as energy, the photon gas has a pressure; the (RELATIVISTIC) pressure of a photon gas is $\boxed{P_\gamma = \varepsilon_\gamma/3.}$ The photon gas within our imaginary box follows the laws of thermodynamics; in particular, the boxful of photons obeys the first law

VOL!

STEPHAN BOLTZM

PRESSURE ≃ ENERGY DENSITY

EINSTEIN:

$V = r^3(z) = \left(\frac{r_0}{1+z}\right)^3$

$\varepsilon_\gamma = \alpha T^4(z) = \alpha T_0^4 (1+z)^4$

IN/OUT = INT + WORK

$$dQ = dE + P\,dV, \quad = 0 \tag{2.37}$$

COSMIC SB-LAW (z)

25

where dQ is the amount of heat flowing into or out of the photon gas in the volume V, dE is the change in the internal energy, P is the pressure, and dV is the change in volume of the box. Since, in a homogeneous universe, there is no net flow of heat (everything is the same temperature, after all), $dQ = 0$. Thus, the first law of thermodynamics, applied to an expanding homogeneous universe, is

$\boxed{P \simeq \frac{E}{V} = \frac{mc^2}{V}}$ ⇓ $\boxed{P = \rho c^2}$

⟺ PRESSURE ≡ ENERGY DENSITY!

(2.37): $dE + PdV = 0$ ⟹
$$\frac{dE}{dt} = -P(t)\frac{dV}{dt}. \tag{2.38}$$

↑ EINSTEIN'S MAIN PREMISE IN G.R.

Since, for the photons of the CMB, $E = \varepsilon_\gamma V = \alpha T^4 V$ and $P = P_\gamma = \alpha T^4/3$, Equation 2.38 can be rewritten in the form

DIFF!
$$\alpha\left(4T^3\frac{dT}{dt}V + T^4\frac{dV}{dt}\right) = -\frac{1}{3}\alpha T^4\frac{dV}{dt}, \tag{2.39}$$

MULT BOTH SIDES BY $\frac{1}{4\alpha T^4 V}$

or

$T(z) = T_0\cdot(1+z)$?
$V(z) = V_0\cdot(1+z)^{-3}$ ⟸
$$\boxed{\frac{1}{T}\frac{dT}{dt} = -\frac{1}{3V}\frac{dV}{dt}.} \tag{2.40}$$

However, since $V \propto a(t)^3$ as the box expands, this means that the rate of change of the photons' temperature is related to the rate of expansion of the universe by the relation

$a \equiv \frac{1}{1+z}$

$T(t) = T_0\cdot(1+z)$ ⟸ $T(t) = \frac{1}{a(t)}$ ⟸
$$\frac{d}{dt}(\ln T) = -\frac{d}{dt}(\ln a). \tag{2.41}$$
⟹ $T(z) = T_0\cdot(1+z)$

COSMIC $T(z)$

This implies the simple relation $T(t) \propto a(t)^{-1}$; the temperature of the cosmic background radiation has dropped by a factor of 1090 since the universe became transparent, because the scale factor $a(t)$ has increased by a factor of 1090 since then. What we now see as a cosmic microwave background was once, at the time the universe became transparent, a cosmic *near infrared* background, with a temperature comparable to that of a relatively cool star like Proxima Centauri.

$= T_0$
$T(z=0) = 2.73K$
$T(z) = T_0(1+z)$
$T(z=1090) \simeq 3000K$
⟺ $p^+ + e^- \Rightarrow H$

The evidence cited so far can all be explained within the framework of a *Hot Big Bang* model, in which the universe was originally very hot and very dense, and since then has been expanding and cooling. The remainder of this book will be devoted to working out the details of the Hot Big Bang model that best fits the universe in which we live.

$V = r^3(z) = \left(\frac{r_0}{1+z}\right)^3$

$\varepsilon_\gamma = \alpha T^4(z) = \alpha T_0^4\cdot(1+z)^4$

COSMIC SB-LAW(z)

H/W 2.4 SHOW/VERIFY THAT
$T(z) = T_0(1+z)^1$ ←
$V(z) = r_0^3(1+z)^3$ ←
$\varepsilon_\gamma(z) = \alpha T_0^4(1+z)^4$ ←

Exercises

2.1 Assume you are a perfect blackbody at a temperature of $T = 310\,\mathrm{K}$. What is the rate, in watts, at which you radiate energy? (For the purposes of this problem, you may assume you are spherical.)

2.2 Since you are made mostly of water, you are very efficient at absorbing microwave photons. If you were in intergalactic space, how many CMB

$z = 1$, for illustration.

From: M. Rowan-Robinson "Cosmology" (Oxford Physics Series)

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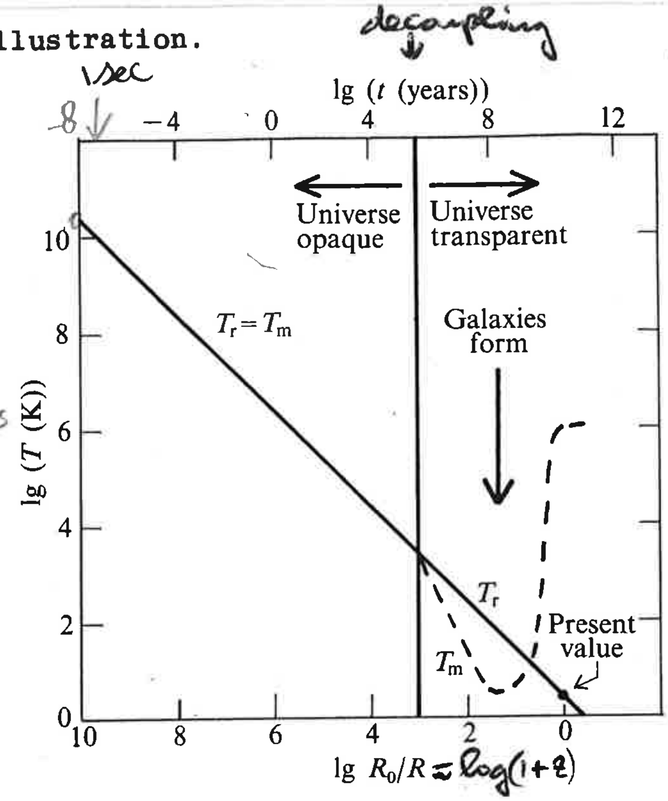


$$T = T_0(1+z)$$

$$T_0 \simeq 2.726$$

$$T(t) \simeq 10^{10}\,K\left(\frac{t}{1\,sec}\right)^{-1/2}$$

Eq. (9.1)

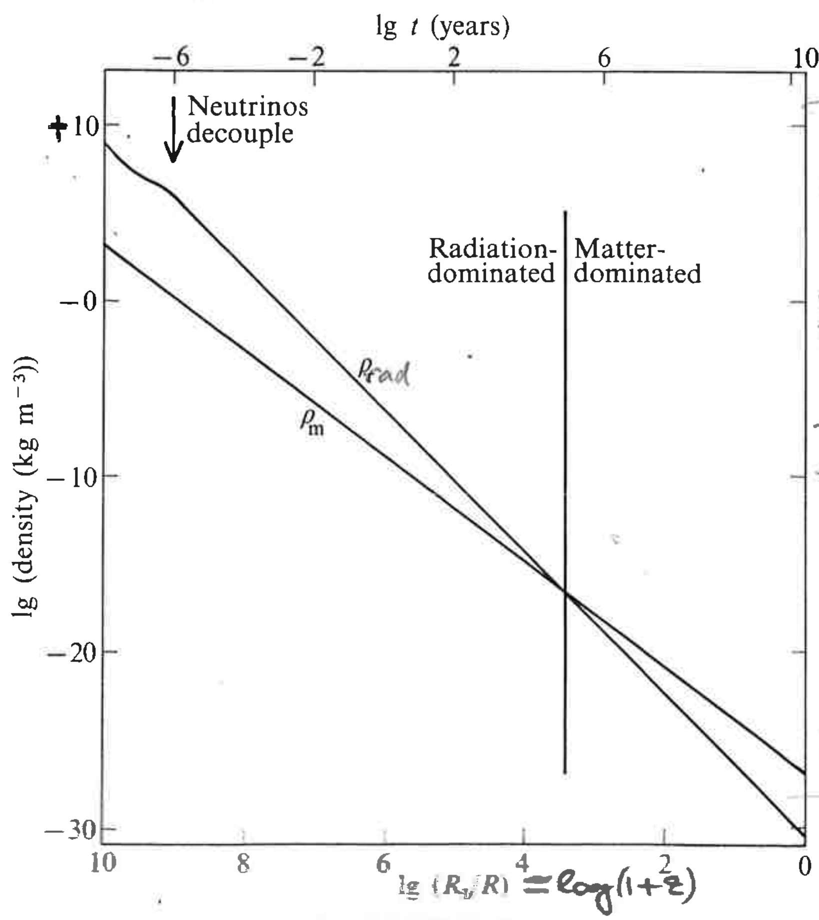
$$\frac{R(t)}{R_0} = a(t) = \frac{1}{1+z}$$

$$\Downarrow$$

$$\frac{R_0}{R(t)} = (1+z) = a^{-1}(t)$$

FIG. 5.3. The variation of the temperature of matter and radiation with epoch.

$z = R(t_0)/R(t) = 3000/2{\cdot}7 \sim 1000.$



$$\rho_m(t) = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3(t)} = \frac{\rho_0}{R^3(t)/R_0^3}$$

$$\Rightarrow \rho_m(t) = \rho_0(1+z)^3$$

$$\rho_{rad} = \varepsilon_\gamma T^4(t)$$

$$= \varepsilon_\gamma T_0^4 (1+z)^4$$

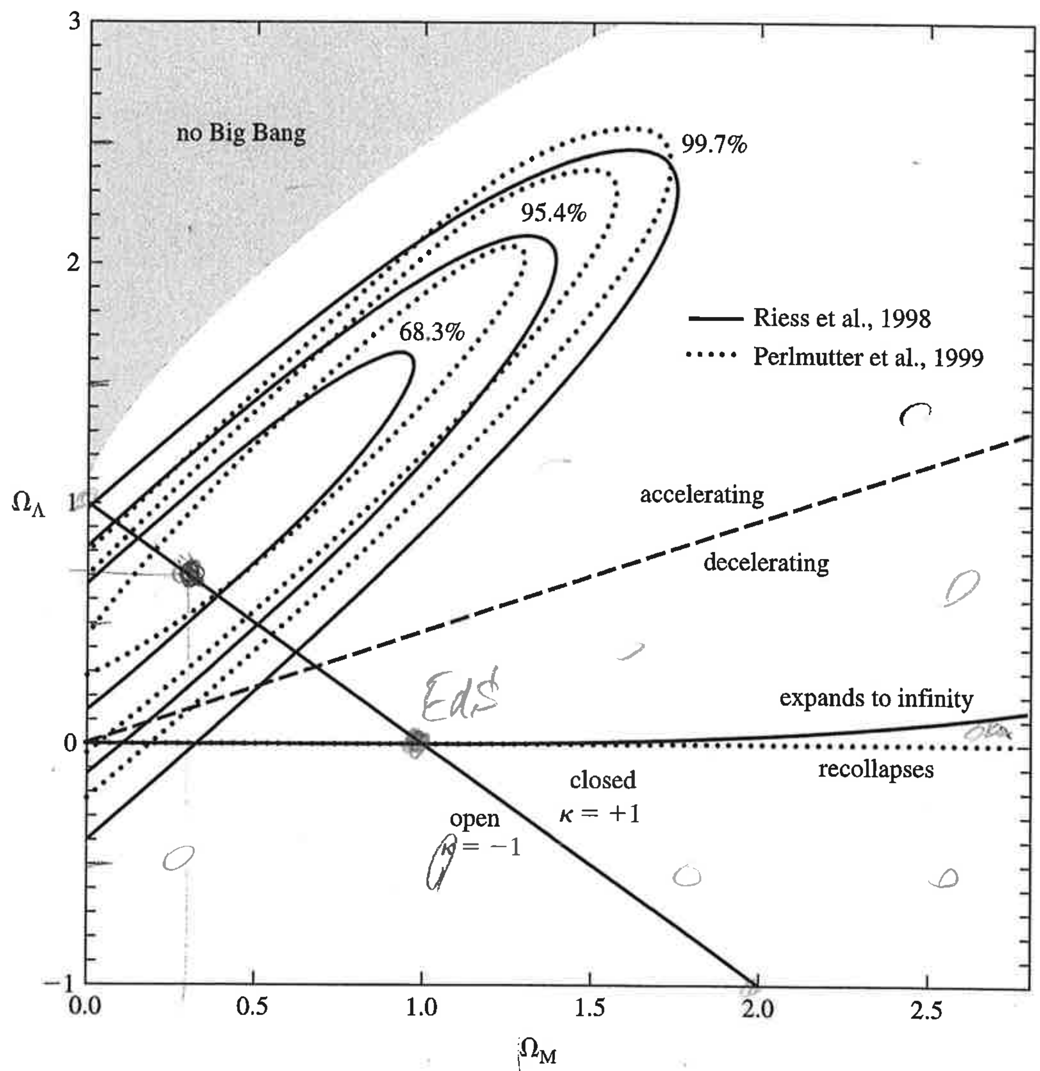
$$\Downarrow$$

$$\rho_r = \rho_{r,0} \cdot (1+z)^4$$

e

FIG. 5.2. The variation of the density of matter and radiation with epoch.

The supernova data extend out to $z \sim 1$; this is beyond the range where an expansion in terms of H_0 and q_0 is adequate to describe the scale factor $a(t)$. Thus, the two supernova teams customarily describe their results in terms of a model universe that contains both matter and a cosmological constant. After choosing values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$, they compute the expected relation between $m - M$ and z, and compare it to the observed data. The results of fitting these model universes are given in Figure 7.6. The ovals drawn on Figure 7.6 enclose those values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ that give the best fit to the supernova data. The results of the two teams (the solid ovals and dotted ovals) give very similar results. Three concentric ovals are shown for each team's result; they correspond to 1σ, 2σ, and 3σ confidence intervals, with the inner oval representing the highest probability.



HP (or TP)

FIGURE 7.6 The values of $\Omega_{m,0}$ (horizontal axis) and $\Omega_{\Lambda,0}$ (vertical axis) that best fit the data shown in Figure 7.5. The solid ovals show the best-fitting values for the High-z Supernova Search Team data; the dotted ovals show the best-fitting values for the Supernova Cosmology Project data.