Introduction

Cosmology is the study of the universe, or cosmos, regarded as a whole. Attempting to cover the study of the entire universe in a single volume may seem like a megalomaniac's dream. The universe, after all, is richly textured, with structures on a vast range of scales; planets orbit stars, stars are collected into galaxies, galaxies are gravitationally bound into clusters, and even clusters of galaxies are found within larger superclusters. Given the complexity of the universe, the only way to condense its history into a single book is by a process of ruthless simplification. For much of this book, therefore, we will be considering the properties of an idealized, perfectly smooth, model universe. Only near the end of the book will we consider how relatively small objects, such as galaxies, clusters, and superclusters, are formed as the universe evolves. It is amusing to note in this context that the words *cosmology* and *cosmetology* come from the same Greek root: the word *kosmos*, meaning harmony or order. Just as cosmetologists try to make a human face more harmonious by smoothing over small blemishes such as pimples and wrinkles, cosmologists sometimes must smooth over small "blemishes" such as galaxies.

A science that regards entire galaxies as being small objects might seem, at first glance, very remote from the concerns of humanity. Nevertheless, cosmology deals with questions that are fundamental to the human condition. The questions that vex humanity are given in the title of a painting by Paul Gauguin (Figure 1.1): "Where do we come from? What are we? Where are we going?" Cosmology grapples with these questions by describing the past, explaining the present, and predicting the future of the universe. Cosmologists ask questions such as "What is the universe made of? Is it finite or infinite in spatial extent? Did it have a beginning some time in the past? Will it come to an end some time in the future?"

Cosmology deals with distances that are very large, objects that are very big, and timescales that are very long. Cosmologists frequently find that the standard SI units are not convenient for their purposes: the meter (m) is awkwardly

Figure 1.1 *Where Do We Come From? What Are We? Where Are We Going?* Paul Gauguin, 1897–98. [Museum of Fine Arts, Boston]

1 pc = 3.26 ly
1 AU = 1.5×10^{8} km = $\frac{1}{206265}$ pc = $\frac{2\pi}{360\times3600}$ pc
1 pc = 3.1×10^{16} m
1 pc = 3.1×10^{13} km
1 Mpc = 3.1×10^{19} km
1 Gpc = 3.1×10^{22} km
1 $M_\odot = 2\times10^{33}$ gr $= 2\times10^{30}$ kg
1 $L_\odot = 4\times10^{33}\,\frac{\text{ergs}}{\text{s}} = 4\times10^{26}$ W (=J/s)
1 $m_H = 1.67\times10^{-27}$ kg $= 1.67\times10^{-24}$ gr
($N_{Av} = \frac{1}{m_H} = 6\times10^{23}$ H-atoms/gr)
1 day = 86,400 s
1 yr = 365.2422 d
1 yr = 3.16×10^{7} s
1 [illegible] = 3.16×10^{16} s
1 J = 6×10^{18} eV = 6×10^{12} MeV

short, the kilogram (kg) is awkwardly tiny, and the second (s) is awkwardly brief. Fortunately, we can adopt the units that have been developed by astronomers for dealing with large distances, masses, and times.

One distance unit used by astronomers is the astronomical unit (AU), equal to the mean distance between the Earth and Sun; in metric units, 1 AU = 1.50×10^{11} m. Although the astronomical unit is a useful length scale within the solar system, it is small compared to the distances between stars. To measure interstellar distances, it is useful to use the parsec (pc), equal to the distance at which 1 AU subtends an angle of 1 arcsecond; in metric units, 1 pc = 3.09×10^{16} m. For example, we are at a distance of 1.30 pc from Proxima Centauri (a small, relatively cool star that is the Sun's nearest neighboring star); we are at a distance of 8500 pc from the center of our galaxy, the Milky Way Galaxy. Although the parsec is a useful length scale within our galaxy, it is small compared to the distances between galaxies. To measure intergalactic distances, we use the megaparsec (Mpc), equal to 10^6 pc, or 3.09×10^{22} m. For example, we are at a distance of 0.76 Mpc from M31 (otherwise known as the Andromeda galaxy) and 15 Mpc from the Virgo cluster (the nearest big cluster of galaxies).

The standard unit of mass used by astronomers is the solar mass ($M_\odot$); in metric units, the Sun's mass is $1\,M_\odot = 1.99 \times 10^{30}$ kg. The total mass of our galaxy is not known as accurately as the mass of the Sun; in round numbers, though, it is $M_{gal} \sim 10^{12}\,M_\odot$. The Sun, incidentally, also provides the standard unit of power used in astronomy. The Sun's luminosity (that is, the rate at which it radiates away energy in the form of light) is $1\,L_\odot = 3.83 \times 10^{26}$ watts. The total luminosity of our galaxy is not known as accurately as the luminosity of the Sun; a good estimate, though, is $L_{gal} \approx 3 \times 10^{10}\,L_\odot$.

For times much longer than a second, it is convenient to use the year (yr) as a unit of time, with $1\,\text{yr} \approx 3.16 \times 10^7$ s. In a cosmological context, a year is frequently an inconveniently short period of time, so cosmologists often use megayears (Myr), with 1 Myr = 10^6 yr = 3.16×10^{13} s. Even longer timescales call for use of gigayears (Gyr), with 1 Gyr = 10^9 yr = 3.16×10^{16} s. For example, the age of the Earth is more conveniently written as 4.57 Gyr than as 1.44×10^{17} s.

$m_e c^2 = 0.511$ MeV
$m_p c^2 = 938$ MeV

In addition to dealing with very large things, cosmology also deals with very small things. Early in its history, as we shall see, the universe was very hot and dense, and some interesting particle physics phenomena were occurring. Consequently, particle physicists have plunged into cosmology, introducing some terminology and units of their own. For instance, particle physicists tend to measure energy units in electron volts (eV) instead of joules (J). The conversion factor between electron volts and joules is 1 eV = 1.60×10^{-19} J. The rest energy of an electron, for instance, is $m_e c^2 = 511\,000$ eV = 0.511 MeV, and the rest energy of a proton is $m_p c^2 = 938.27$ MeV $= 1836.1 m_e c^2$.

$\frac{m_{p^+}}{m_{e^-}} \simeq 1836.1$

1 J = 6×10^{18} eV / 0.511 MeV $\simeq 1.2 \times 10^{13}\, m_e c^2$ $\simeq 6.4 \times 10^{9}\, m_p c^2$

When you stop to think of it, you realize that the units of meters, megaparsecs, kilograms, solar masses, seconds, and gigayears could only be devised by ten-fingered Earthlings obsessed with the properties of water. An eighteen-tentacled silicon-based lifeform from a planet orbiting Betelgeuse would probably devise a different set of units. A more universal, less culturally biased system of units is the Planck system, based on the universal constants G, c, and $\hbar$. Combining the Newtonian gravitational constant, $G = 6.67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$, the speed of light, $c = 3.00 \times 10^8\ \mathrm{m\,s^{-1}}$, and the reduced Planck constant, $\hbar = h/(2\pi) = 1.05 \times 10^{-34}$ J s $= 6.58 \times 10^{-16}$ eV s, yields a unique length scale, known as the Planck length:

NEWTON: $F_G = \frac{GM.m}{R^2}$

EINSTEIN S.R. $E = m.c^2$

Planck Q.M. $E = h.\nu = \frac{hc}{\lambda}$

$\hbar = \frac{h}{2\pi}$

(≠ PROVE)

$\frac{\ell_P}{t_P} \equiv c$!

$$\ell_P \equiv \left(\frac{G\hbar}{c^3}\right)^{1/2} = 1.62 \times 10^{-35}\ \mathrm{m}. \tag{1.1}$$

HW 1.2 SHOW THAT $r_{S,U} = \frac{2GM_P}{c^2} = \frac{2G}{c^2}\left(\frac{\hbar c}{G}\right)^{1/2} = 2G^{1/2}\hbar^{1/2}/c^{3/2} = 2\ell_P$!

The same constants can be combined to yield the Planck mass,[1]

$$M_P \equiv \left(\frac{\hbar c}{G}\right)^{1/2} = 2.18 \times 10^{-8}\ \mathrm{kg}, \tag{1.2}$$

HW.1 * $\rho_P = \frac{E_P}{V_P} = \frac{E_P}{\ell_P^3}$

and the Planck time,

HW 1.1 $\rho_P = \frac{E_P}{\ell_P^3} = \frac{M_P c^2}{\ell_P^3} = \frac{\hbar^{1/2} c^{1/2} G^{-1/2} . c^2}{(G\hbar/c^3)^{3/2}} = \hbar^{-1} . c^{5/2} . c^{9/2} . G^{-2} = \hbar^{-1} c^7 G^{-2}$ $\Rightarrow \rho_P = \frac{c^7}{\hbar G^2}$

$\left(\rho_P = \frac{E_P}{\ell_P^3}\right) . t_P^2 = \frac{c^2}{G}$

$$t_P \equiv \left(\frac{G\hbar}{c^5}\right)^{1/2} = 5.39 \times 10^{-44}\ \mathrm{s}. \tag{1.3}$$

Using Einstein's relation between mass and energy, we can also define the Planck energy,

$\boxed{\rho_P = \hbar^{-1} . c^7 . G^{-2}} = \frac{M_P c^2}{\ell_P^3} = \frac{\hbar^{1/2} c^{1/2} G^{-1/2} c^2}{(G\hbar/c^3)^{3/2}}$

$$E_P = M_P c^2 = 1.96 \times 10^9\ \mathrm{J} = 1.22 \times 10^{28}\ \mathrm{eV}. \tag{1.4}$$

By bringing the Boltzmann constant, $k = 8.62 \times 10^{-5}\ \mathrm{eV\,K^{-1}}$, into the act, we can also define the Planck temperature,

$k = 1.38 \times 10^{-23}$ J K^{-1}

* $E_P = k.T_P = M_P c^2$

$$T_P = E_P/k = 1.42 \times 10^{32}\ \mathrm{K} = \frac{M_P . c^2}{k} \tag{1.5}$$

[1] The Planck mass is roughly equal to the mass of a grain of sand a quarter of a millimeter across.

$E_P^2 = (kT_P)^2 = (M_P . c^2)^2 = \frac{\hbar . c^5}{G}$

$R_H = R_0$
$R_0 = 13.8$ Glyr
$\simeq 4.3$ Gpc
$\simeq 13 \times 10^{22}$ km
$\simeq 1.3 \times 10^{26}$ m

$\frac{R_H}{R_e} = \frac{1.3 \times 10^{26} \text{ m}}{10^{-15} \text{ m}} \simeq 10^{41}$!

$\frac{R_H}{\ell_P} \simeq \frac{1.3 \times 10^{26} \text{ m}}{1.6 \times 10^{-35} \text{ m}} \simeq 10^{61}$!

GXY/4π sr $M_\odot$/GXY
1/12
$M_U \simeq 10 \times 10^{11} M_\odot$
$\simeq 10^{23} M_\odot$

$1 M_\odot = 2 \times 10^{33}$ gr
$= 2 \times 10^{33}$ gr / 1.67×10^{-24} gr/H
$\simeq 1.2 \times 10^{57}$ H atoms

$\Rightarrow M_{UNIVERSE}$
$M_U = 10^{23} \times 10^{57}$ H
$\simeq 10^{80}$ H ATOMS!

$M_U \simeq 10^{80} \simeq \left(\frac{R_H}{r_e}\right)^2$
$\simeq \left(\frac{R_H}{\ell_P}\right)^{4/3}$
!!

When distance, mass, time, and temperature are measured in the appropriate Planck units, then $c = k = \hbar = G = 1$. This is convenient for individuals who have difficulty in remembering the numerical values of physical constants. However, using Planck units can have potentially confusing side effects. For instance, many cosmology texts, after noting that $c = k = \hbar = G = 1$ when Planck units are used, then proceed to omit c, k, $\hbar$, and/or G from all equations. For instance, Einstein's celebrated equation, $E = mc^2$, becomes $E = m$. The blatant dimensional incorrectness of such an equation is jarring, but it simply means that the rest energy of an object, measured in units of the Planck energy, is equal to its mass, measured in units of the Planck mass. In this book, however, I will retain all factors of c, k, $\hbar$, and G, for the sake of clarity.

Here we will deal with distances ranging from the Planck length to 10^4 Mpc or so, a span of some 61 orders of magnitude. Dealing with such a wide range of length scales requires a stretch of the imagination, to be sure. However, cosmologists are not permitted to let their imaginations run totally unfettered. Cosmology, I emphasize strongly, is based ultimately on observation of the universe around us. Even in ancient times, cosmology was based on observations; unfortunately, those observations were frequently imperfect and incomplete. Ancient Egyptians, for instance, looked at the desert plains stretching away from the Nile valley and the blue sky overhead. Based on their observations, they developed a model of the universe in which a flat Earth (symbolized by the earth god Geb in Figure 1.2) was covered by a solid dome (symbolized by the sky goddess Nut). Underneath the sky dome, the disk of the Sun was carried from east to west by the sun god Ra. Greek cosmology was based on more precise and sophisticated observations. Ancient Greek astronomers deduced, from their observations, that the Earth and

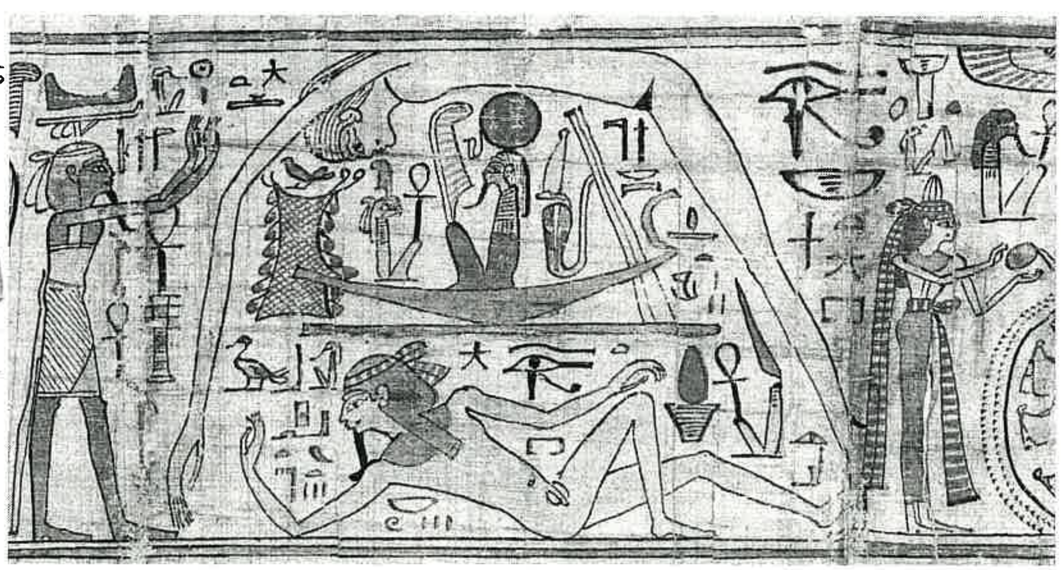

Figure 1.2 The ancient Egyptian view of the cosmos: the sky goddess Nut arches over the earth god Geb, while the sun god Ra travels between them in a reed boat. (Book of the Dead of Nespakashuty, ca. 1000 BC) [Musée du Louvre, Paris]

Moon are spherical, that the Sun is much farther from the Earth than the Moon is, and that the distance from the Earth to the stars is much greater than the Earth's diameter. Based on this knowledge, Greek cosmologists devised a "two-sphere" model of the universe, in which the spherical Earth is surrounded by a much larger celestial sphere, a spherical shell to which the stars are attached. Between the Earth and the celestial sphere, in this model, the Sun, Moon, and planets move on their complicated apparatus of epicycles and deferents.

Although cosmology is ultimately based on observation, sometimes observations temporarily lag behind theory. During periods when data are lacking, cosmologists may adopt a new model for aesthetic or philosophical reasons. For instance, when Copernicus proposed a new Sun-centered model of the universe, to replace the Earth-centered two-sphere model of the Greeks, he didn't base his model on new observational discoveries. Rather, he believed that putting the Earth in motion around the Sun resulted in a conceptually simpler, more appealing model of the universe. Direct observational evidence didn't reveal that the Earth revolves around the Sun, rather than vice versa, until the discovery of the aberration of starlight in the year 1728, nearly two centuries after the death of Copernicus. Foucault didn't demonstrate the rotation of the Earth, another prediction of the Copernican model, until 1851, over *three* centuries after the death of Copernicus. However, although observations sometimes lag behind theory in this way, every cosmological model that isn't eventually supported by observational evidence must remain pure speculation.

The current standard model for the universe is the "Hot Big Bang" model, which states that the universe has expanded from an initially hot and dense state to its current relatively cool and tenuous state, and that the expansion is still going on today. To see why cosmologists have embraced the Hot Big Bang model, let us turn, in the next chapter, to the fundamental observations on which modern cosmology is based.

ASTRONOMICAL QUANTITIES & UNITS

(1) DISTANCES d $\quad (c = 3\times10^8$ m/s $= 3\times10^5$ km/s$)$

$1\,pc = 3.26\,ly \qquad 1\,ly = c.t = 3\times10^8 \times 86{,}400 \times 365.2422 = 9.47\,10^{15}\,m$

$1\,AU = \frac{2\pi}{360\times3600}\,pc = \frac{1}{206{,}265}\,pc = 1.5\times10^8\,km = 1.5\times10^{11}\,m$ $\quad$ (s/day) (d/trop year)

→ $1\,pc \cong 3.09\times10^{16}\,m = 3.09\times10^{13}\,km$

$1\,Kpc \simeq \quad \simeq 3.1\times10^{16}\,km$

$1\,Mpc = \quad \simeq 3.1\times10^{19}\,km$ $\qquad$ TRUE AGE OF UNIV

$1\,Gpc = \quad \simeq 3.1\times10^{22}\,km$ $\qquad$ $\simeq 13.8$ Gyr

$R_{HUBBLE} \simeq 4.38\,Gpc \simeq 1.35\times10^{23}\,km \simeq 1.35\times10^{26}\,m \simeq 14\,Glyr$

(2) TIME t $\quad (c = d/t = 3\times10^8$ m/s$)$

$1\,day = 60\times60\times24 = 86{,}400\,sec$

$1\,yr = 365.2422\,d = 3.16\times10^7\,s$

$1\,Myr = \quad = 3.16\times10^{13}\,s$

$1\,Gyr = \quad = 3.16\times10^{16}\,s$

$t_{HUBBLE} \simeq 13.8\,Gyr \cong 4.35\times10^{17}\,s$

(3) MASSES

$1\,m_H = 1.67\times10^{-27}\,kg = 1.67\times10^{-24}\,gr \Leftarrow$ $\left(N_{AVOGADRO} = 1/m_H \simeq 6\times10^{23}\,H/gr\right)$

$1\,M_\odot \simeq 2\times10^{30}\,kg = 2\times10^{33}\,gr \simeq 1.2\times10^{57}$ H-ATOMS

$M_{UNIV} \simeq \underset{(GXY/4\pi sr)}{10^{12}} \times \underset{(M_\odot/GXY)}{10^{11}} \simeq 10^{23}\,M_\odot \simeq 10^{23}\times1.2\times10^{57} \simeq 10^{80}\,m_H$

(4) ENERGY $\quad$ (1 WATT = 1 J/s)

(eV is energy to move one e^- over 1 Volt)

$1\,JOULE = 6\times10^{18}\,eV = 6\times10^{12}\,MeV$

$m_{p^+} \simeq m_H \simeq 1.67\times10^{-27}\,kg \Rightarrow m_p c^2 \simeq 938\,MeV$

$m_{e^-} \simeq \frac{1}{1836}\,m_{p^+} \simeq 9.1\times10^{-31}\,kg \Rightarrow m_{e^-} c^2 \simeq 0.511\,MeV$

$1\,J \simeq 6\times10^{18}\,eV \simeq 1.2\times10^{13}\,m_e c^2 \simeq 6.4\times10^{9}\,m_p c^2$

ASTRONOMICAL LARGE NUMBERS

HUBBLE RADIUS $R_H = R_0 \simeq 13.8\,\mathrm{Glyr} \simeq 4.38\,\mathrm{Gpc}$

$\simeq 1.35 \times 10^{23}\,\mathrm{km} = 1.35 \times 10^{26}\,\mathrm{m}$

EDDINGTON'S LARGE NUMBERS

a) $$\frac{R_H}{r_e} \simeq \frac{1.3 \times 10^{26}\,\mathrm{m}}{10^{-15}\,\mathrm{m}} \simeq 10^{-41} \;!$$

b) $$\frac{R_H}{\ell_{PL}} \simeq \frac{1.3 \times 10^{26}\,\mathrm{m}}{1.6 \times 10^{-35}\,\mathrm{m}} \simeq 10^{61} \quad \left[\simeq \sqrt{\frac{\Lambda_{PLANCK}}{\Lambda_{TODAY}} \simeq 10^{123}} \right]$$

c) $$M_{UNIVERSE} = M_U \simeq 10^{23} \frac{M_\odot}{4\pi\,\mathrm{sr}} \times 10^{57} \frac{N_H}{M_\odot} \simeq 10^{80} \text{ H-ATOMS!}$$

$$\Longrightarrow \quad M_U \simeq 10^{80} N_H \simeq \left(\frac{R_H}{r_e}\right)^2 \simeq \left(\frac{R_H}{\ell_{PL}}\right)^{4/3} \simeq \left[\left(\frac{\Lambda_{PL}}{\Lambda_{z=0}}\right)^{2/3} \right]$$